Modeling Marine Electromagnetic Survey with Radial Basis Function Networks

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Abstract. A marine electromagnetic survey is an engineering endeavour to discover the location and dimension of a hydrocarbon layer under an ocean floor. In this kind of survey, an array of electric and magnetic receivers are located on the sea floor and record the scattered, refracted and reflected electromagnetic wave, which has been transmitted by an electric dipole antenna towed by a vessel. The data recorded in receivers must be processed and further analysed to estimate the hydrocarbon location and dimension. To conduct those analyses successfully, a radial basis function (RBF) network could be employed to become a forward model of the input-output relationship of the data from a marine electromagnetic survey. This type of neural networks is working based on distances between its inputs and predetermined centres of some basis functions. A previous research had been conducted to model the same marine electromagnetic survey using another type of neural networks, which is a multi layer perceptron (MLP) network. By comparing their validation and training performances (mean-squared errors and correlation coefficients), it is concluded that, in this case, the MLP network is comparatively better than the RBF network. 1

Keywords: controlled source electromagnetic method; forward modeling; multilayer perceptron; radial basis function.

1 Introduction

Marine electromagnetic survey is an engineering endeavour to remotely determine the location and dimension of a hydrocarbon (i.e. oil, gas, basalt, or hydrate) layer inside a seabed. There are many techniques to do this kind of survey, such as by seismic sounding, well-borehole logging, and controlled source electromagnetic (CSEM) method. In the last method, an electric dipole antenna, while submerged and deep-towed by a ship, is emitting electromagnetic signals throughout the sea and its surrounding in various directions.

1 This manuscript is an extended version of our previous paper, entitled Radial Basis Function Networks for Modeling Marine Electromagnetic Survey, which had been presented on 2011 International Conference on Electrical Engineering and Informatics, 17-19 July 2011, Bandung, Indonesia.
At the same time, an array of electromagnetic receivers is located at the bottom of the sea (Figure 1). These receivers record the electric and magnetic fields which are directed, refracted or reflected from all parts of air, seawater, sediments, and hydrocarbon layer.

![Figure 1](image1.png)

**Figure 1** Conceptual diagram of the marine CSEM method. A deep-towed transmitter close to the seafloor injects a current of several hundred amps into the seawater from an electric dipole, creating magnetic and electric fields that propagate diffusively into the seafloor. Dipole receivers record the seafloor electric fields at various ranges from the transmitter (adapted from [1]).

To process the recorded data, several steps should be taken, similar to the flowchart of Figure 2. First, the electromagnetic field measurement is modified to be suitable for the next steps and resulted in some observation data. Meanwhile, a model of seabed structure is assumed to be supplied to forward modeling step. This step will produce some predictive data that will be compared to the observation data. Misfits, between these two sets of data, become an indicator to decide whether the model can be accepted. If the misfits are considered un-acceptable, the inversion step should be taken to modify the original model and then supplied again to the step of forward modeling. Therefore, forward modeling is an essential procedure in finding the acceptable model, which then can be used to achieve the goal of seabed logging.

Forward modeling is performed to acquire a representation of the real physical phenomena. The result of this effort is a mathematical model which is sufficiently suitable to describe the original phenomena. Also, with forward modeling, a set of predictive or synthetic data can be generated that can be compared to the real measurement data.
According to Baan and Jutten [2], artificial neural networks have been applied successfully to a variety geophysical problem. In this domain, neural networks have been used for waveform recognition and first-break picking; for electromagnetic, magnetotelluric, and seismic inversion purposes; for shear-wave splitting, well-log analysis, trace editing, seismic deconvolution, and event classification; and for many other problems.

Figure 2 The flowchart of steps to be taken to determine an acceptable model of a seabed resistive structure. Furthermore, this accepted model can be used to determine the location of the hydrocarbon layer inside a seabed.

To generate data in the form of input and target pairs for the RBF network training, a simulation software package, such as COMSOL Multiphysics, can be utilized. Multiphysics is interactive software for modeling and solving all kinds of scientific and engineering problems based on partial derivative equations (PDEs). With this package, conventional models can easily be extended from one type of physics into multiphysics models that solve coupled physics phenomena. To solve the PDEs, Multiphysics uses the proven finite element
method (FEM). The software runs the finite element analysis together with adaptive meshing and error control using a variety of numerical solvers. There are optional modules provided for several key application areas, such as electromagnetism and radio frequency (RF). The RF Module contains a set of application modes adapted to a broad category of electromagnetic simulations [3],[4].

The objective of this research is to confirm that a radial basis function (RBF) network could be used to model a CSEM survey. Then, this modeling result will be compared to the result of a previous research that had been conducted to model the same marine electromagnetic survey using another type of neural networks, which is a multi layer perceptron (MLP) network.

2 Methodology

The base model for this study is the canonical 1D reservoir that was considered previously in Constable and Weiss (2006), and is shown in Figure 3. This model consists of a 100-ohm-m resistive hydrocarbon reservoir of 100-m thickness located 1 km beneath the seafloor, with surrounding 1-ohm-m sediments. The conductive seawater is 1 km deep, and the transmitter (dipole antenna) is located 25 m above the seafloor.

![Figure 3](image)

Figure 3 The canonical 1D structure of seabed with an oil/gas reservoir between 1-ohm-meter sediments of overburden and underburden [1].

In this research, some adjustments have been done to the canonical model above. At the present time, the seabed model is not including the hydrocarbon reservoir so that this model can be used later as a reference to be compared to the other models which include some hydrocarbon layers. Also, the twenty-one receivers are assumed to be located on the seafloor and inline with the direction of the transmitter movement.
2.1 Generating Data

Generally, there are five main steps to generate data using Multiphysics and its modules [3]; which are preliminary, drawing, physics, calculation, and post-processing steps. Each step will be further explained, while at the same time a structure of seabed is being built concurrently, according to the previously mentioned canonical model.

2.1.1 Preliminary Step

At the beginning of running the Multiphysics software, a template can be chosen that is appropriate to the model which is being built. In this research, the work-space dimension is 3D and the suitable application mode is Harmonic Propagation which is found under Electromagnetic Waves option, inside the RF Module.

In the main window, all constants that will be used in this modeling can be defined as in Table 1. The constant values were taken from references [1],[3], and [5].

<table>
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<th>Name</th>
<th>Expression</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
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<td>80</td>
<td>Relative permittivity of seawater</td>
</tr>
<tr>
<td>rho1</td>
<td>$0.3 , [\text{ohtm} \cdot \text{meter}]$</td>
<td>$0.3 , [\Omega \cdot \text{m}]$</td>
<td>Resistivity of seawater</td>
</tr>
<tr>
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<td>$1/\text{rho1}$</td>
<td>$3.333333 , [\text{S} / \text{m}]$</td>
<td>Conductivity of seawater</td>
</tr>
<tr>
<td>eps2</td>
<td>30</td>
<td>30</td>
<td>Relative permittivity of sediment</td>
</tr>
<tr>
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<td>$1 , [\Omega \cdot \text{m}]$</td>
<td>Resistivity of sediment</td>
</tr>
<tr>
<td>sig2</td>
<td>$1/\text{rho2}$</td>
<td>$1 , [\text{S} / \text{m}]$</td>
<td>Conductivity of sediment</td>
</tr>
<tr>
<td>freq</td>
<td>$1 , [\text{Hz}]$</td>
<td>$1 , [1/\text{s}]$</td>
<td>Transmitter frequency</td>
</tr>
<tr>
<td>txi</td>
<td>$10e3 , [\text{A}]$</td>
<td>$10000 , [\text{A}]$</td>
<td>Transmitter current</td>
</tr>
</tbody>
</table>

2.1.2 Drawing Step

In this research, the horizontal length of the seabed model is 22 km, and the vertical length of all layers are in accord with the 1D canonical structure in Figure 3 (except without the hydrocarbon layer). All layers can be drawn using Rectangle/Square object which is provided in the tools bar of Multiphysics. The result of this step is displayed in Figure 4.
In Figure 4, the drawing of a transmitter and a line of receivers have been achieved using the Line object. The transmitter TX is 100-m long and the receiver RX are assumed to be continuous along the 20-km line on top of the seafloor. Recorded data of an electric field component are sampled at 21 locations (0, 1, 2, ..., 20 km) along the receiver RX.

**Figure 4** The 3D structure of seabed model with frequency = 0.5 Hz, and TX location at 5000 m. The thickness of the ‘slab’ is 2 km in the z-axis direction.

### 2.1.3 Physics Step

To define the types of materials which construct a model structure, submenu Subdomain Settings is chosen and each subdomain can be specified for its relative permittivity ($\varepsilon_r$), electric conductivity ($\sigma$), and relative permeability ($\mu_r$). In this research, for every layer of the model (seawater and sediment), its properties are set according to the already defined Constants (Table 1).

The condition for every boundary surface can be specified using submenu Boundary Settings. For the model built in this research, the whole exterior surfaces that enclose the model structure are set to scattering boundary condition with the spherical wave type.

For the transmitter and receiver, their properties can be defined by choosing submenu Edge Settings. The edge setting for the transmitter is current in edge segment direction with value of $t_{xi}$ (see Table 1); this is the same as $I_0 = 10$ kA. For the receiver, its edge setting is perfect electric conductor; while for all of the remaining edges, their edge setting are continuity.

The working frequency of this model would be varied from 0.1 Hz until 1 Hz; and each of this value can be applied in Multiphysics by choosing submenu Scalar Variables.
2.1.4 Calculation Step

After all those initial definitions, drawings, and element specifications, the mesh for finite element computation can be determined. In this research, the mesh is customized with the maximum element size is equal to 1000. The result of this mesh making is displayed in Figure 5.

![Figure 5](image)

**Figure 5** The mesh of the seabed model with working frequency = 0.5 Hz, and TX location at 5000 m.

The actual finite element calculation is started by choosing submenu Solve Problem. While running its computation, the software is displaying the progress of its calculation. After a certain duration (an average of 1 minute in this research), the result of problem solving is displayed as a 3D figure of a subdomain plot. (This type of figure, actually, can be chosen differently by using submenu Plot Parameters).

2.1.5 Post-processing Step

From various choices in the Postprocessing menu, the Line/ Extrusion option is chosen to obtain a graph of electric field amplitudes versus the receivers’ positions, as displayed in Figure 6. In addition, this step is also used to export data of the electric fields, in a complex number format, which are sampled from 21 locations of the receivers.

All those data generation steps (detailed description about them can be found in Arif, et al. [6]) had been repeated to simulate the movement of the dipole antenna in 21 positions (0, 1, 2, …, 20 km) and the 7 variations of working frequency (0.01, 0.05, 0.1, 0.25, 0.5, 0.75, and 1 Hz). In the end, the result was 3087 values of electric field amplitudes, in a format of real and imaginary parts, which later will be used to train and test the RBF network as a model of marine CSEM survey.
2.2 Radial Basis Function Networks

A RBF network is one of the important classes of artificial neural networks, in which the activation of hidden neurons are determined by the *distance* between the input vector and a prototype vector, which is also called the centre vector. This activation is quite different from the one in hidden neurons of a MLP network, which computes a non-linear function of the scalar product of the input vector and a weight vector.

![Figure 6](image)

**Figure 6** The electric field amplitude versus receiver position for the seabed model with frequency = 0.5 Hz and the transmitter TX location at 5000 m.

One of the consequences of this difference is the training of RBF networks can be substantially faster than the method used to train the MLP networks. This follows from a two-stage training procedure: In the first stage, the parameters controlling the basis functions (corresponding to neurons in a hidden layer) are determined using relatively quick, unsupervised methods (i.e. methods which use only the input data and not the target data). The second stage then involves the determination of the final layer weight, which requires the solution of a linear problem, and which therefore also fast.

In Figure 7, each hidden neuron consists of a basis function. The lines connecting basis function \( \phi_j \) to the input neuron \( x_i \) represent the corresponding elements \( \mu_{ji} \) of the centre vector \( \mu_j \). The weights \( w_j \) are shown as lines from the hidden neurons to the output neuron \( y \), and the bias \( w_0 \) is shown as weight from an extra 'basis function' \( \phi_0 \) whose output is fixed at 1. This bias will compensate
for the differences between the average value over the data set of the basis function activations and the corresponding average value of the targets [7].

Based on this architecture, the formula for the RBF network mapping between its inputs and output can be written as

\[ y(x) = \sum_{j=0}^{M} w_j \phi_j(x) + w_0 \]  

(1)

Figure 7  Architecture of a RBF network with three neurons in the input layer and one neuron in the output layer, while neurons in the hidden layer can be varied in number (adapted from [7]).

For the case of Gaussian basis function,

\[ \phi_j(x) = \exp\left( -\frac{\|x - \mu_j\|^2}{2\sigma_j^2} \right) \]  

(2)

where \( x \) is the 3-dimensional input vector with elements \( x_i \), \( \mu_j \) is the vector specifying the centre of basis function \( \phi \) and has elements \( \mu_{ji} \), and \( \sigma_j \) is the width parameter of basis function \( \phi_j \).

In the first stage of the RBF network training, the input data set alone is used to determine the parameters of the basis functions (e.g. \( \mu_j \) and \( \sigma_j \) for the spherical Gaussian basis function considered above). Instead of simply choosing a subset of the data points as the basis function centres, a clustering technique can be used to find a set of centres \( \mu_j \) which more accurately reflects the distribution of the data points.
In this research, *K-means clustering algorithm* is selected to determine the basis function centres. If there are *N* data points *x*\(^n\) in total, then the algorithm will find a set of *K* representative vectors \(\mu_j\) where \(j = 1, \ldots, K\). The algorithm seeks to partition the data points \(\{x^n\}\) into *K* disjoint subsets \(S_j\) containing *N*\(_j\) data points, in such a way to minimize the sum-of-squares clustering function given by

\[
J = \sum_{j=1}^{K} \sum_{n \in S_j} \|x^n - \mu_j\|^2
\]  

(3)

where \(\mu_j\) is the mean of the data points in set \(S_j\) and is given by

\[
\mu_j = \frac{1}{N_j} \sum_{n \in S_j} x^n
\]  

(4)

In this research, the width parameter \(\sigma_j\) is kept constant for all \(j = 1, \ldots, M\).

After determining the basis functions, they are then kept fixed while the second-layer weights are found in the second stage of training. To begin this stage, the bias parameter in Eq. (1) is absorbed into the weights to give

\[
y(x) = \sum_{j=0}^{M} w_j \phi_j(x)
\]  

(5)

and can be written in matrix notation as

\[
y(x) = W \phi
\]  

(6)

where \(W = (w_j)\) and \(\phi = (\phi_j)\).

The weights \(W\) can be determined by minimization of a suitable error function. In this case, it is convenient to consider a sum-of-squares error function given by

\[
E = \frac{1}{2} \sum_n \left( y(x^n) - t^n \right)^2
\]  

(7)

where \(t^n\) is the target value for the output neuron when the RBF network is presented with input vector \(x^n\). Since the error function is a quadratic function of the weights, its minimum can be found in terms of the solution of a set of linear equations:

\[
\Phi^T \Phi W = \Phi^T T
\]  

(8)

where \((T)_n = t^n\) and \((\Phi)_{n j} = \phi_j(x^n)\). The formal solution for the weight is given by [7]

\[
W^T = \Phi^T \Phi
\]  

(9)
where the notation $\Phi^\dagger$ denotes the pseudo-inverse of $\Phi$ and is given by
\[
\Phi^\dagger \equiv (\Phi^T \Phi)^{-1} \Phi^T
\] (10)

Based on all above equations, the RBF network has been constructed using MATLAB programming with 3 neurons in the input layer. These neurons act as source nodes to supply the various values of frequency, transmitter and receiver positions of the SBL model. The one neuron in the output layer represents the electric field amplitude value. While neurons in the hidden layer would be varied between 1 and 20 neurons.

The data set that has been generated with Multiphysics was further processed to convert the complex number format of the electric field amplitude values into their corresponding magnitude and phase values. Then, only the normalized logarithmic magnitude values were used in the training and testing of the RBF network. From 3087 values, 3000 values were used in the training session, while the remaining 87 values were used to test the network.

3 Results and Discussion

The implementation of the RBF network was realized using a MATLAB script. The commands used to define, train, and test the network were formulated from the appropriate equations in Methodology section. When the training was over, the resulted network was validated using the same data that were used in the training process. One of the results of this validation is shown in Figures 8 and 9.

Figure 8 Validation of the RBF network with 10 neurons in its hidden layer. Linear regression between 3000 values of output and target.
Figure 9 Validation of the RBF network with 10 neurons in its hidden layer. Comparison of the first 300 values of electric field amplitude between output (circle) and target (line).

After that validation, the RBF network is further tested using another data set which was different from the training data. The result of this testing is shown in Figures 10 and 11.

Figure 10 Testing of the RBF network with 10 neurons in its hidden layer. Linear regression between 87 values of output and target.
All these training, validation and testing were conducted for various configurations of the RBF networks. The difference aspect of these configurations was the neurons number in the hidden layer, which is the same as the centres number of each network. The performances of these various RBF networks can be seen in Table 2. Based on this table, it can be inferred that the RBF network with better performances is the one with 10 neurons in its hidden layer. Although other RBF networks with more neurons in their hidden layer have the similar performances, but it is more efficient to choose the network with less neurons in its hidden layer when the performances are comparable.

Number of hidden neurons or basis functions also affected the computation time to do the training of the RBF network. The last column of Table 2 displays an exponentially growth of the computation time as the centres number increasing linearly. This phenomenon can be seen more clearly in the bar graph of Figure 12.

In the previous work (see Arif, et al. [8]), a MLP network had been used to model the same electromagnetic survey as in this research. The optimum MLP architecture has 10 neurons in its hidden layer with validation performances: (MSE = 0.001028, R = 0.9787); and testing performances: (MSE = 0.001195, R = 0.99066). These indicators are comparatively better than the optimum RBF network performances, which is for validation: (MSE = 0.001702, R = 0.96451); and testing: (MSE = 0.003298, R = 0.98466). This difference is probably caused by the better ability of the MLP network in approaching the target peaks comparing to the RBF network (see Figures 9 and 11).
Table 2  Performance of RBF networks.

<table>
<thead>
<tr>
<th>Centres</th>
<th>Validation Perform. MSE</th>
<th>Reg ‘R’</th>
<th>Testing Perform. MSE</th>
<th>Reg ‘R’</th>
<th>Comp. time (s)</th>
</tr>
</thead>
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<td>0.051708</td>
<td>-0.23086</td>
<td>0.128150</td>
<td>-0.07043</td>
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<tr>
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</table>

Figure 12  Computation time of the training is affected by number of centres of the RBF network.
4 Conclusions

Based on the results of this research, it was shown that a RBF network, after several appropriate training and testing, has a possibility to become a model for marine electromagnetic survey with CSEM method.

By comparing the performances of several RBF networks with different number of hidden neurons (or basis functions with their own centres), the best RBF network to model the CSEM survey is the one with 10 neurons in its hidden layer. But the performances of this optimum RBF network are still slightly lower when it is compared with the previous built MLP network to model the same survey.

In future work, the RBF network can be improved using a localized version of this network. Then, it could be used to generate the electric field amplitude magnitude at a certain working frequency and at a particular location of the transmitter and receiver of a marine electromagnetic survey.

Nomenclature

\[ E = \text{error function} \]
\[ J = \text{clustering function} \]
\[ j = \text{index of basis function} \]
\[ K = \text{number of clusters or disjoint subsets} \]
\[ M = \text{number of basis functions} \]
\[ N = \text{number of data points} \]
\[ n = \text{index of data point} \]
\[ N_j = \text{number of data points in disjoint subset} \]
\[ S_j = \text{disjoint subset} \]
\[ T = \text{vector of target values} \]
\[ t^n = \text{target value} \]
\[ W = \text{matrix of weights} \]
\[ w_j = \text{weight from hidden layer to output layer} \]
\[ w_0 = \text{bias in hidden layer} \]
\[ x = \text{3-dimensional input vector} \]
\[ x_i = \text{neuron in input layer} \]
\[ x^n = \text{data point} \]
\[ y = \text{neuron in output layer} \]
\[ \Phi = \text{matrix of basis functions} \]
\[ \phi = \text{matrix of centres} \]
\[ \phi_j = \text{basis function} \]
\( \phi_0 \) = extra basis function
\( \mu_j \) = vector of centres
\( \mu_{ji} \) = element of centre vector
\( \sigma_j \) = width parameter

References


