Linear Parameter Varying Versus Linear Time Invariant Reduced Order Controller Design of Turboprop Aircraft Dynamics

Widowati¹, Bambang Riyanto² & Hari Muhammad³

¹Mathematics Department, Diponegoro University

Jl. Prof. H. Soedarto, S.H., Semarang, 50275, Indonesia

²Electrical Engineering Study Program, Institut Teknologi Bandung

Jl. Ganesha No. 10 Bandung, 40132, Indonesia

³Aerospace Engineering Study Program, Institut Teknologi Bandung

E-mail: widowati_math@undip.ac.id

Abstract. The applicability of parameter varying reduced order controllers to aircraft model is proposed. The generalization of the balanced singular perturbation method of linear time invariant (LTI) system is used to reduce the order of linear parameter varying (LPV) system. Based on the reduced order model the low-order LPV controller is designed by using H_∞ synthesis technique. The performance of the reduced order controller is examined by applying it to lateral-directional control of aircraft model having 20th order. Furthermore, the time responses of the closed loop system with reduced order LPV controllers and reduced order LTI controller is compared. From the simulation results, the 8th order LPV controller can maintain stability and to provide the same level of closed-loop systems performance as the full-order LPV controller. It is different with the reduced-order LTI controller that cannot maintain stability and performance for all allowable parameter trajectories.

Keywords: H_∞ synthesis; singular perturbation approximation; reduced-order LPV controller; stability; aircraft dynamics.

1 Introduction

The generalization of the balanced truncation method of LTI systems to reduce the model order of LPV systems has been published by Zobaidi, et al. [1], Goddard [2], and Wood, et al. [3]. Next, Widowati, et al. [4] have used a generalization balanced truncation to reduce the order of a parameter varying controller. In paper [4], we have investigated a measure of the degradation of the parameter varying closed-loop performance due to the reduced-LPV controller. Balanced singular perturbation approximation (BSPA) to reduce the controller order of LTI system has been published by Saragih and Yoshida [5]. Further, Widowati, et al. [6] proposed a method to reduce the unstable LPV systems by generalizing BSPA for LTI systems via Contractive Right Coprime

Factorizations. Furthermore, Saragih and Widowati [7] have presented sufficient conditions for the existence of a right coprime factorization of parameter varying controller.

In this paper we compare the performance of the LPV closed loop system with reduced order LPV controllers and reduced order LTI controller. In comparison with method proposed in [1], this paper uses balanced singular perturbation method, whereas in [1] balanced truncation method was used to reduce a high order-plant. To verify the validity of the proposed method it is applied to model reduction for the N-250 aircraft model. The dynamic of an aircraft may be particularly severe in case of high performance aircraft that fly over a wide range of operational flight conditions (take-off, cruise, landing, altitude, airspeed). In the early days of automatic flight control systems, most of the systems were designed using LTI control. However, aircraft dynamics vary for different flight conditions. Therefore, LTI controller is only valid for a certain flight condition and cannot guarantee performance for other flight conditions. If a LTI controller is not capable of maintaining performance, then controller parameter values needs some adjustment. It is well known that variation of some aircraft parameters is strongly related to the air data variables such as airspeed and altitude of the aircraft. It is necessary to rely scheduling of the flight controller parameters on the air data variables, a technique referred to as gain-scheduling.

In an attempt to reduce conservatism of the control design and to improve the numerical computation for systems significantly affected by measurable time varying parameter, several new Linear Parameter Varying (LPV) approaches have emerged in the last few years. LPV approaches explicitly take into account the relationship between real-time parameter variations and the control system stability and performance. Therefore, the LPV controller theoretically guarantees performance and robustness for whole ranges of operating conditions. Most of the LPV approaches are based upon Linear Matrix Inequalities (LMI) and are solved numerically with some efficiency. While multitudes of theoretical results exist in this area [8-11, 2], only a few aerospace application oriented papers have been reported to demonstrate effectiveness of the LPV control [12, 13]. Related results by authors of the current paper on application of LPV to flight control appeared in [14, 15], but these works focused on full order controller case. The purpose of this paper is to apply robust gain scheduling for uncertain LPV systems to lateral-directional control of N-250 aircraft by using reduced order controller.

The paper is organized as follows. Section 2 describes a brief review of the linear parameter varying (LPV) systems. The LPV synthesis is given in Section 3. Order reduction of LPV system by using BSPA method is discussed in

Section 4. In Section 5, the reduced LTI and LPV control design are demonstrated for a lateral-directional motion of aircraft dynamics. Finally, concluding remarks are presented in Section 6.

2 Brief Review of LPV System

In this paper, we consider linear parameter varying systems. For a compact subset $W \subset R^s$, the parameter variation set F_ρ denotes the set of all piecewise continuous mapping $R(\text{time})$ into W with a finite number of discontinuity in any interval, $F_\rho = \{\rho: R \rightarrow W, \rho_{t_{\min}} \leq \rho_i \leq \rho_{t_{\max}}, i = 1, 2, \dots, s\}$.

A compact set $W \subset R^s$, along with continuous functions $A: R^s \rightarrow R^{n \times n}$, $B: R^s \rightarrow R^{n \times n_u}$, $C: R^s \rightarrow R^{n_y \times n}$, $D: R^s \rightarrow R^{n_y \times n_u}$ represent an n th-order parameter varying plant, $G(\rho)$, whose dynamics evolve as

$$\begin{aligned} \dot{x}(t) &= A(\rho(t))x(t) + B(\rho(t))u(t), \\ y(t) &= C(\rho(t))x(t) + D(\rho(t))u(t), \end{aligned} \quad (1)$$

where $x(t) \in R^n$, $y(t) \in R^{n_y}$, $u(t) \in R^{n_u}$. A state space realization of the parameter varying plant, $G(\rho)$, is written as

$$G(\rho) = \left[\begin{array}{c|c} A(\rho(t)) & B(\rho(t)) \\ \hline C(\rho(t)) & D(\rho(t)) \end{array} \right], \quad \forall \rho(t) \in F_\rho.$$

The parameters of LPV system (1) are the functions of parameter ρ with unbounded parameter variation rates; hence the dependence of system parameters from $\dot{\rho}$ can be omitted.

The parameter varying systems $G(\rho)$ is a quadratically stable [2] if there exists a real positive-definite matrix $P = P^T > 0$ such that

$$A^T(\rho(t))P + P A(\rho(t)) < 0, \quad \forall \rho(t) \in F_\rho \quad (2)$$

The induced L_2 norm of a quadratically stable parameter varying systems, $G(\rho)$, with zero initial conditions, is defined as [2]

$$\|G(\rho)\|_{i,2} = \sup_{\rho(t) \in F_\rho} \sup_{u \neq 0, u \in L_2} \frac{\|y\|_2}{\|u\|_2}. \quad (3)$$

3 LPV Synthesis

To formulate a performance oriented parameter varying output feedback synthesis problem, the parameter varying plant is written as follows

$$\begin{aligned} \dot{x}(t) &= A(\rho(t))x(t) + B(\rho(t))u(t) + D(\rho(t))w(t), \\ y(t) &= C(\rho(t))x(t) + D(\rho(t))w(t), \end{aligned} \quad (4)$$

where y is the measurement, $u(t)$ is the control input, $w(t)$ is the disturbance, $z(t)$ is the controlled output, $x(t)$ is the state vector. Readers are referred to references [2, 3] for the definition of the quadratic stabilizability and quadratic detectability. The following assumptions are made to parameter varying plant.

1. $D_{22}(\rho(t)) = 0$
2. $B_2(\rho(t))$, $C_2(\rho(t))$, $D_{12}(\rho(t))$, and $D_{21}(\rho(t))$ are parameter-independent.
3. The pairs $(A(\rho(t)), B_2)$ and $(A(\rho(t)), C_2)$ are quadratically stabilizable and quadratically detectable over W respectively.

A construction of full-order controller has been developed by Apkarian, et.al [8]. The design objectives are to satisfy H_∞ performance criteria, i.e., the parameter varying closed-loop systems is a quadratically stable over W and the L_2 gain of the parameter varying closed-loop systems is bounded by γ , $\gamma > 0$ for all possible trajectories ρ . We assume that the full-order controller has an m th-order. The parameter dependent full-order controller has state space realization as follows. For brevity, t is omitted.

$$K(\rho) = \begin{bmatrix} \bar{A}_K & \bar{B}_K \\ \bar{C}_K & \bar{D}_K \end{bmatrix}, \quad \forall \rho \in F_\rho. \quad (5)$$

The parameters of the gain-scheduled controller given by (5) are the functions of parameter ρ with unbounded parameter variation rate, hence the dependence of controller parameters from $\dot{\rho}$ can be omitted.

Theorem 3.1. [8] Consider the generalized LPV plant (4) with parameter trajectories. There exists a gain scheduled output feedback controller (5) enforcing internal stability and a bound γ on the L_2 gain of the closed-loop system, whenever there exist symmetric matrices X and Y and a parameter dependent quadruple of state-space data $(\bar{A}_K, \bar{B}_K, \bar{C}_K, \bar{D}_K)$ such that the

following linear matrix inequality problem holds, (for brevity, the parameter ρ dependence of state space matrices is omitted).

$$\begin{bmatrix} XA + \bar{B}_K C_2 + (*) & * & * & * \\ \bar{A}_K^T & AY + \bar{B}_2 \bar{C}_K + (*) & * & * \\ (XB_1 + \bar{B}_K D_{21})^T & (B_1 + B_2 \bar{D}_K D_{21})^T & -\gamma I & * \\ C_1 + D_{12} \bar{D}_K C_2 & C_1 Y + D_{21} \bar{C}_K & D_{11} + D_{12} \bar{D}_K D_{21} & -\gamma I \end{bmatrix} < 0, \quad (6)$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \succ 0. \quad (7)$$

Note that the notation $(*)$ is induced by symmetry, for example

$$\begin{bmatrix} M + N + (*) & * \\ K & L \end{bmatrix} = \begin{bmatrix} M + M^T + N + N^T & K^T \\ K & L \end{bmatrix}$$

In such a case, a gain scheduled (parameter varying) controller of the form (5) is readily obtained with the following two step scheme:

1. Solve for N, M , the factorization problem: $I - XYNN^T$
2. Compute A_K, B_K, C_K, D_K with

$$\begin{aligned} A_K &= A - B_K (C_K D_K)^{-1} C_K A \\ B_K &= B - B_K (C_K D_K)^{-1} C_K B \\ C_K &= C - C_K (D_K D_K)^{-1} D_K C \\ D_K &= D - C_K (D_K D_K)^{-1} D_K D \end{aligned}$$

4 Order Reduction of LPV System

In this section, we use the generalization of the BSPA method to reduce the model order of the parameter varying systems (1). This generalization is based on solutions of the parameter varying Lyapunov inequalities of the LPV system. Consider an n -order quadratically stable model of LPV systems in equation (1). By using a balancing state transformation matrix we obtain the transformed controllability and observability Gramians.

$$\begin{aligned} \tilde{P} &= P - P Q (P Q + Q P)^{-1} P \\ \tilde{Q} &= Q - Q P (P Q + Q P)^{-1} Q \end{aligned} \quad \text{and}$$

$$\sigma_j = \sqrt{\lambda_j(PQ)}, \sigma_j \geq \sigma_{j+1}, j = 1, \dots, r, r+1, \dots, n.$$

P and Q are solutions of Lyapunov inequalities [2].

A balanced LPV system can be expressed by

$$\begin{aligned} \dot{\tilde{x}} &= \tilde{A}(\rho)\tilde{x} \\ y(t) &= \tilde{C}(\rho)x(t) + \tilde{D}(\rho)u(t). \end{aligned} \quad (8)$$

Partition the balanced LPV system conformably with $\Sigma = \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix}$ is described as follows

$$\begin{aligned} \begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \end{bmatrix} &= \begin{bmatrix} A_{11}(\rho) & A_{12}(\rho) \\ A_{21}(\rho) & A_{22}(\rho) \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \begin{bmatrix} B_1(\rho) \\ B_2(\rho) \end{bmatrix} u(t), \\ y(t) &= \begin{bmatrix} C_1(\rho) & C_2(\rho) \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \tilde{D}(\rho)u(t), \end{aligned} \quad (9)$$

where $\tilde{x}_1 \in R^r$ and $\tilde{x}_2 \in R^{n-r}$.

Furthermore, the BSPA method is used to reduce the order of the balanced LPV systems (9). The dynamic of the reduced r -order model can be expressed in the form

$$\begin{aligned} \dot{x}_1(t) &= \bar{A}(\rho)x_1(t) + \bar{B}(\rho)u(t), \\ y(t) &= \bar{C}(\rho)x_1(t) + \bar{D}(\rho)u(t), \end{aligned} \quad (10)$$

where

$$\begin{aligned} \bar{A}(\rho) &= A_{11}(\rho) - A_{12}(\rho)A_{22}^{-1}(\rho)A_{21}(\rho), \\ \bar{B}(\rho) &= B_1(\rho) - A_{12}(\rho)A_{22}^{-1}(\rho)B_2(\rho), \\ \bar{C}(\rho) &= C_1(\rho) - C_2(\rho)A_{22}^{-1}(\rho)A_{21}(\rho), \\ \bar{D}(\rho) &= \tilde{D}(\rho) - C_2(\rho)A_{22}^{-1}(\rho)B_2(\rho), \end{aligned} \quad (11)$$

by assuming $A_{22}(\rho)$ non singular $\forall \rho \in F_\rho$.

If the LPV system not quadratically stable, but must be quadratically stabilizable and detectable then it cannot be directly reduced by using the above method. If the unstable LPV system is stabilizable via a parameter varying state feedback, then we can construct a quadratically stable Contractive Right Coprime Factorization (CRCF). Consider the following theorem.

Theorem 4.1. [3] Let $G(\rho)$ have a continuous, quadratically stabilizable, quadratically detectable realizations, then contractive right graph symbol of $G(\rho)$ is given by

$$G(\rho) := \begin{bmatrix} N(\rho) \\ M(\rho) \end{bmatrix} = \left[\begin{array}{c|c} \frac{A(\rho) + B(\rho)F(\rho)}{C(\rho) + D(\rho)F(\rho)} & \frac{B(\rho)S^{-1/2}(\rho)}{D(\rho)S^{-1/2}(\rho)} \\ \hline F(\rho) & S^{-1/2}(\rho) \end{array} \right], \quad (12)$$

where $F(\rho) = -S^{-1}(\rho)B(\rho)X + D^T(\rho)C(\rho)$, $S(\rho) = I + D^T(\rho)D(\rho)$,

$R(\rho) = I + D(\rho)D^T(\rho)$, and $X = X^T > 0$ is a constant solution of the Generalized Control Riccati Inequality (GCRI)

$$(A(\rho) - B(\rho)S^{-1}(\rho)D^T(\rho)C(\rho))^T X + X(A(\rho) - B(\rho)S^{-1}(\rho)D^T(\rho)C(\rho)) - XB(\rho)S^{-1}(\rho)B^T(\rho)X + C^T(\rho)R^{-1}(\rho)C(\rho) < 0, \forall \rho \in F_\rho. \quad (13)$$

Further, we can reduce the coprime factor realization by using a generalization of the BSPA method [6] with procedure as follows. $G(\rho)$ is written as $G(\rho) = N(\rho)M^{-1}(\rho)$, where $(N(\rho), M(\rho))$ represent a CRCF of $G(\rho)$, $N(\rho)$ and $M(\rho)$ are quadratically stable. Let $Q=X$ and $P=(I+YX)^{-1}Y$ are observability and controllability Gramians of $G(\rho)$, with X solves GCRI and Y solves a Generalized Filtering Riccati Inequality (GFRI),

$$(A(\rho) - B(\rho)D^T(\rho)R^{-1}(\rho)C(\rho))Y + Y(A(\rho) - B(\rho)D^T(\rho)R^{-1}(\rho)C(\rho))^T - YC^T(\rho)S^{-1}(\rho)C(\rho)Y + B(\rho)S^{-1}(\rho)B^T(\rho) < 0, \forall \rho \in F_\rho. \quad (14)$$

Furthermore, $G(\rho)$ is balanced by using a state transformation matrix T such that the transformed Gramians $\tilde{P} = \tilde{Q} = \Sigma$ and then SPA method is applied to obtain

$$G_r(\rho) := \begin{bmatrix} N_r(\rho) \\ M_r(\rho) \end{bmatrix},$$

which has r^{th} -order, $r < n$. Moreover, obtain the reduced-order LPV system,

$$\bar{G}_r(\rho) = N_r(\rho)M_r^{-1}(\rho).$$

5 Computational Issues

The constraints given by linear matrix inequalities (6) and (7) are parameter dependent, i.e., there is an infinite set of linear matrix inequalities, where for every values of parameter. These linear matrix inequalities can be solved by polytopic technique. The parameter vector, $\rho(t)$, takes values in a box of R^s

with corners $\{\rho_{v_i} : i=1,2,\dots,r\}$, $r=2^s$. In the other word, the parameter vector varies in polytope with vertices $\rho_{v_1}, \rho_{v_2}, \dots, \rho_{v_r}$, so that the parameter can be written by convex combination,

$$\rho(t) = Co\{\rho_{v_1}, \rho_{v_2}, \dots, \rho_{v_r}\} := \sum_{i=1}^r \alpha_i \rho_{v_i} : \alpha_i \geq 0, \sum_{i=1}^r \alpha_i = 1, \forall t \geq 0.$$

The problem of linear matrix inequalities (LMIs) (6) and (7) are approximated by solving LMIs at each of the vertices. The constraints given by this LMIs at each of the vertices can be solved using interior point method in convex programming. The consequence of this approximation is that the number of LMIs becomes $2^{s+1} + 1$ and the number of decision variables becomes $n_x(n_x+1)$, with n_x is number of states. The variables X, Y in LMIs (6)-(7) are approximated by convex interpolation of solution at each of the vertices.

The parameter varying controller (5) with the state space matrices $A_K(\rho)$, $B_K(\rho)$, $C_K(\rho)$, and $D_K(\rho)$ range in a polytope of matrices whose vertices are the images of the vertices $\rho_{v_1}, \rho_{v_2}, \dots, \rho_{v_r}$ that is,

$$\begin{bmatrix} A_K(\rho) & B_K(\rho) \\ C_K(\rho) & D_K(\rho) \end{bmatrix} = Co\left\{ \begin{bmatrix} A_{K_i} & B_{K_i} \\ C_{K_i} & D_{K_i} \end{bmatrix}, i=1,2,\dots,r \right\} := \sum_{i=1}^r \alpha_i \begin{bmatrix} A_{K_i} & B_{K_i} \\ C_{K_i} & D_{K_i} \end{bmatrix},$$

$$\alpha_i \geq 0, \sum_{i=1}^r \alpha_i = 1.$$

6 N-250 LPV Control Design and Simulation

To verify the capability of the singular perturbation approximation, simulations are given by applying it to reduce an aircraft model [15]. The performance of the closed-loop systems with reduced order LPV controller and LTI reduced controller will be compared.

6.1 Modeling of N-250 Aircraft

N-250 is a turboprop aircraft which has two engines, and equipped with three-axis fly-by-wire (FBW) system. The FBW system is designed to deflect the aircraft control surface comprising flap, spoiler, rudder, aileron, and T-tail elevator by applying electric signals to actuators (see Figure 1). N-250 is a regional commuter aircraft which has capacity of 64-70 passengers, economic cruise velocity at 300 knot (556 km/hour), maximum velocity of 330 knot, and flight distance of 1.482 km.

Figure 1 N250 Aircraft control surface.

For simulation purpose, N250 aircraft is classified as Class II aircraft where the whole mission is dominated by flight phase B. Consequently, ideal model is chosen from the minimum requirement of Flight Phase B, Class II aircraft. Linear equation of the N-250 PA-2 used in this research is derived from non-linear equation of motion of the aircraft. Aerodynamic forces and moment coefficients were obtained from N-250 PA-2 wind-tunnel test conducted by P.T. Dirgantara Indonesia (formerly PT IPTN). Linearization process of equation of motion is carried out by introducing small perturbation over all variables of the equation, a technique referred to as Small Perturbation Method. In the linearized form, the resulting equation of motion consists of two independent motions: longitudinal and lateral-directional motion. The linearized model is fine tuned using flight test data and parameter identification process. It is based on the flight test data analysis introduced by Laban [16] and Lean [17]. Figure 2 shows a technique of how system identification process was applied to the problem of estimating the aerodynamic coefficients based on flight test data. During the flight test program, a specially-designed dynamic maneuver was applied to the test aircraft, and various aircraft dynamic variables were measured. The aircraft control surface deflections were measured and were applied to aircraft dynamic system model. Basically, this process consists of two major phases (see Figure 2), given the real-time presentation of the raw flight test measurements, which in our case, is provided through a telemetry system at former Flight Test Center,

IPTN. The first phase is Data Reconstruction/Compatibility-Check from measurement data of aircraft response due to input given by the pilot.

Figure 2 N250 Aircraft Identification process.

In data reconstruction process, a matching mechanism is performed on data input and output recorded during flight test, and is based on aircraft kinematics equations. In this process, Extended Kalman Filter is applied to suppress noise which arises due to atmospheric conditions and inaccuracy/uncertainty in flight parameter measurement devices. In the second phase, an optimization is performed to obtain values of aerodynamic coefficients using Least Square method [18] which minimizes the output mismatch. Modeling the N250 aircraft motion based on flight testing took several months, and various flight maneuvers testing should be conducted to cover most of the flight envelope.

The Aircraft primary control surface consists of aileron and rudder for lateral-directional motion and elevator for longitudinal motion. Secondary control surface consists of spoiler and flaps. In this paper, we consider only the lateral-directional dynamics, the longitudinal dynamics are assumed to remain at equilibrium. Lateral-directional dynamic of the aircraft can be expressed as the unstable LPV system with the state-space model as follows.

$$\begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} Y'_v(V, f) & Y'_p(V, f) & Y'_r(V, f) & g \cos \theta & 0 \\ L'_v(V, f) & L'_p(V, f) & L'_r(V, f) & 0 & 0 \\ N'_v(V, f) & N'_p(V, f) & N'_r(V, f) & 0 & 0 \\ 0 & I & \tan \theta & 0 & 0 \\ 0 & 0 & \sec \theta & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ r \\ \phi \\ \psi \end{bmatrix} + \begin{bmatrix} Y'_{da}(V, f) & Y'_{dr}(V, f) \\ L'_{da}(V, f) & L'_{dr}(V, f) \\ N'_{da}(V, f) & N'_{dr}(V, f) \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_A \\ \delta_R \end{bmatrix},$$

$$\begin{bmatrix} \beta \\ p \\ r \\ \phi \\ \psi \\ a_y \end{bmatrix} = \begin{bmatrix} \frac{57,3}{V} & 0 & 0 & 0 & 0 \\ 0 & 57,3 & 0 & 0 & 0 \\ 0 & 0 & 57,3 & 0 & 0 \\ 0 & 0 & 0 & 57,3 & 0 \\ 0 & 0 & 0 & 0 & 57,3 \\ Y''_v(V, f) & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ r \\ \phi \\ \psi \end{bmatrix},$$

where

$$\begin{aligned}
Y'_v(V, f) &= \frac{\zeta u_e S}{2m} C_{y_\beta}(V, f) & Y'_p(V, f) &= \frac{\zeta u_e S}{4m} C_{y_p}(V, f) & Y'_r(V, f) &= \frac{\zeta u_e S}{4m} C_{y_r}(V, f) \\
L'_v(V, f) &= \frac{L_v}{I_{xxs} + I_{xzs} N_v}(V, f) & L'_p(V, f) &= \frac{L_p}{I_{xxs} + I_{xzs} N_p}(V, f) & L'_r(V, f) &= \frac{L_r}{I_{xxs} + I_{xzs} N_r}(V, f) \\
L_v &= \frac{\zeta u_e^2 S b}{2I_{xx}} C_{l_\beta}(V, f) & L_p &= \frac{\zeta u_e^2 S b^2}{4I_{xx}} C_{l_p}(V, f) & L_r &= \frac{\zeta u_e^2 S b^2}{4I_{xx}} C_{l_r}(V, f) \\
N_v &= \frac{\zeta u_e^2 S b}{2I_{zz}} C_{n_\beta}(V, f) & N_p &= \frac{\zeta u_e^2 S b^2}{4I_{zz}} C_{n_p}(V, f) & N_r &= \frac{\zeta u_e^2 S b^2}{4I_{zz}} C_{n_r}(V, f) \\
N'_v(V, f) &= \frac{N_v}{I_{zxs} + I_{xzs} L_v}(V, f) & N'_p(V, f) &= \frac{N_p}{I_{zxs} + I_{xzs} L_p}(V, f) & N'_r(V, f) &= \frac{N_r}{I_{zxs} + I_{xzs} L_r}(V, f) \\
Y'_{da}(V, f) &= \frac{Y_{da}}{m}(V, f) & Y'_{dr}(V, f) &= \frac{Y_{dr}}{m}(V, f) & L'_{da}(V, f) &= \frac{L_{da}}{I_{xxs} + I_{xzs} N_{da}}(V, f) \\
L'_{dr}(V, f) &= \frac{L_{dr}}{I_{xxs} + I_{xzs} N_{dr}}(V, f) & N'_{da}(V, f) &= \frac{N_{da}}{I_{zxs} + I_{xzs} L_{da}}(V, f) \\
Y''_v(V, f) &= \frac{Y_v V}{mg}(V, f) & N'_{dr}(V, f) &= \frac{N_{dr}}{I_{zxs} + I_{xzs} L_{dr}}(V, f)
\end{aligned}$$

in which state vector consists of lateral velocity(v), roll rate (p), yaw rate (r), roll angle(ϕ), and azimuth angle(ψ); input vector consists of aileron deflection (δ_A) and rudder deflection (δ_R); measurement vector consists of side-slip (β), roll rate (p), yaw rate (r), roll angle(ϕ), azimuth angle(ψ), and lateral acceleration(a_y), ζ : air density, g : gravity, m : mass of the aircraft, S : surface area of the aircraft, b : wing span. For the detail of the symbols refer to Etkin [19]. For the aircraft control, all states are available for feedback. The dynamics

of the N-250 aircraft under consideration vary greatly as a function of speed (V) and flaps setting (f). Signal V and f are measured in real time. The take-off aircraft speed ranges between 80 KEAS(Knot Equivalent Air Speeds) and 320 KEAS. The flaps setting vary in 0, 20, 30, and 40 deg. The nominal data of the aircraft is given in Table 1.

Table 1 The nominal data of the aircraft at take-off flight conditions.

Parameters	values
Speed (knots) : KEAS	80 - 320
Center of Gravitation(%) :cg	26.7
Aircraft Mass : kg	20267.5
Altitude : feet	1250
Flap setting : deg	0, 20, 30, 40

The parameter dependent state space data are obtained by trimming and linearizing the non linear model at speeds $V = \{80, 85, 90, 95, 100, 105, 110, 115, 120, 125, 130, 135, 140, 145, 150, 155, 160, 170, 175, 180, 185, 190, 195, 200, 205, 210, 215, 220, 225, 230, 235, 240, 245, 250, 255, 260, 265, 270, 275, 280, 285, 290, 295, 300, 305, 310, 315, 320\}$ KEAS and flap setting $\{0, 20, 30, 40\}$ deg. Next, we choose entries of parameter dependent state space data as the vertices of the polytopic plant. However, choosing all the entries as the vertices would results in large number of LMI constrains to be checked and can cause computational impractical. Hence, we consider only 5 entries of state-space data which are most significantly affected by the aircraft speed and flaps setting. By observing singular values plots of the plants with respect to variation of state-space data which are affected by the true aircraft speed and flaps setting, it turns out that entries $L'_v, L'_p, N'_v, N'_p, N'_r$ provides significant changes on the plant. The variation V and f result in parameters range of $\rho_1 = L'_v \in [-0.3132, -0.0181]$, $\rho_2 = L'_p \in [-5.6951, -1.2095]$, $\rho_3 = N'_v \in [-0.0220, 0.0237]$, $\rho_4 = N'_p \in [-1.4561, 0.9450]$, and $\rho_5 = N'_r \in [-1.1020, -0.2057]$. The high-order plant consists of the lateral-directional dynamic and weighting functions which are incorporated to satisfy the controller design specifications. The high order plant of aircraft model has 20th order.

6.2 Reduced Order Controller Design

Reduced-order model is calculated by using BSPA method which is suggested in the preceding section. Based on the reduced order models the low-order controllers are designed by using H_∞ synthesis. Note that the parameters $L'_v, L'_p, N'_v, N'_p, N'_r$ enter the state space matrices in an affine way and are

considered as representing a convex polytope with $32 = 2^5$ vertices. Allow the parameters to be time varying with no rate bound which make it necessary to find solutions of the LMIs (6) and (7) evaluated at each of the 32 vertices. The X , Y solutions of these LMIs are approximated by convex interpolation of solution at each of the vertices.

Furthermore, the full-order and reduced-order LPV controller's states space data were constructed using convex interpolation of their values at the 32 vertices. The upper bound of the parameter varying closed-loop performance for all variations parameters is 50.0003. The LMI Control Toolbox for MATLAB [20] is used for numerical computations. The aim of the controller design is to satisfy the following specifications.

1. Frequency domain specification: the closed-loop system has bandwidth of 10 rad/s, low sensitivity in frequency ≤ 8 rad/s, in high frequency range the measurement noise is attenuated around 2 dB.
2. Time domain specification: Steady state errors are within 7% tolerance, overshoot ≤ 10 % and transient response in 4-8 seconds, control surface magnitude and rates do not exceed actuator saturation limits, system response to command meets level 1 flying quality.

Table 2 Maximum deflection and rate of aileron and rudder.

	Maximum deflection	Maximum rate
Aileron	± 22 degree	50 degree/second
Rudder	± 20 degree	37 degree/second

The controlled output consists of aileron position, rudder position, aileron rate, rudder rate, side-slip error, and yaw-rate error. In this case, to simulate the time responses of the gain-scheduled controller along parameter trajectories, we choose the spiral parameter trajectories,

$$\rho_1 = \dot{L}_v = -0.12685 + 0.1007 \exp(-4t) \cdot \cos(100t)$$

$$\rho_2 = \dot{L}_p = -2.8805 + 1.71195 \exp(-4t) \cdot \sin(100t)$$

$$\rho_3 = \dot{N}_v = 0.0041 + 0.0173 \exp(-4t) \cdot \sin(100t)$$

$$\rho_4 = \dot{N}_p = -0.15395 + 0.760 \exp(-4t) \cdot \cos(100t)$$

$$\rho_5 = \dot{N}_r = -0.5832 + 0.287 \exp(-4t) \cdot \sin(100t)$$

Figure 3 Time responses of the closed-loop system with the 20th- LPV controllers and LTI controller for all parameter trajectories.

The time responses of the closed-loop system with the 20th-order (full-order) LPV and fixed LTI controllers are described in Figure 3. The LTI controller is design for fixed parameter values of the aircraft flight conditions at speed = 90 KEAS, center of gravity = 26.7 %, mass = 20,267.5 kg, altitude = 1250 feet, and flap setting = 40 degree.

Next, we compare the parameter varying closed-loop performance with the full-order and reduced-order LPV controller and LTI controller. Time responses of the closed-loop system with the 20th-, 10th-, 9th-, and 8th-order LPV controllers for all parameter trajectories with respect to impulse input are given in Figure 4. From Figure 3 and Figure 4, it can be seen that the closed-loop time responses achieve stability within 6 seconds except the closed-loop with LTI controller time responses of the aileron and rudder position achieve stability within 7.5 seconds.

The step responses have a settling time of less than 8 seconds. These Figures also show that all saturation and rate limits for aileron and rudder are satisfied the design specifications.

Figure 4 Time responses of the closed-loop system with the full-order and reduced-order LPV controllers for all parameter trajectories.

The time responses of the parameter varying closed-loop systems with the reduced (19th)-order LTI controllers are depicted in Figure 5. From Figure 5, it can be seen that the closed-loop time responses cannot be made stable. From above results, the order of the LPV plant found by BSPA method can be reduced as low as 8th while maintaining the closed-loop system stability and providing the same level of the closed-loop system performance as the 20th - order LPV controller.

Figure 5 Time responses of the closed-loop system with the 19th-order LTI controller for all parameter trajectories.

7 Concluding Remarks

The turboprop aircraft model having 20th-order has been reduced by using balanced singular perturbation method. The effectiveness of the H_∞ controller based on the reduced order model to control lateral-directional motion of aircraft was proposed. A reduced LTI controller cannot maintain performance for all allowable parameter trajectories. Therefore, we have constructed reduced LPV controllers which guarantee performance and robustness for whole ranges of operating conditions. The reduced order LPV controller is able to do better over the entire operating range of the turboprop aircraft than reduced order LTI controller.

Acknowledgement

We would like to thank the reviewers for their detailed comments and suggestions.

References

- [1] El-Zobaidi, H.M.H. & Jaimoukha, I., *Robust Control and Model and Controller Reduction of Linear Parameter Varying Systems*, Proc. of the 37th IEEE Conference on Decision and Control, Tampa, Florida USA **3**, pp. 3015-3020, 1998.
- [2] Goddard, P.J., *Performance-Perserving Controller Approximation*, PhD Dissertation, Trinity College Cambridge, 1995.
- [3] Wood, D., Goddard, P.J. & Glover, K., *Aproximation of Linear Parameter Varying System*, Proceedings of 35th IEEE Conference on Decision and Control, Kobe, Japan, 1996.
- [4] Widowati, Nababan, S.M., Riyanto, B., Saragih, R., *Reduced-order of LPV Control with Guaranteed Closed-loop Performance*, Journal of Indonesian Mathematics Society, **12**(1), 2006.
- [5] Saragih, R. & Yoshida, K., *Reduced-Order of Structural Control using Singular Perturbation Approach*, Proc. of the 4th Int. Conf. on MOVIC, **3**, pp. 539-544, 1998.
- [6] Widowati, Riyanto, B., Saragih, R., Nababan, S.M., *Model Reduction for Unstable LPV Systems Based on Coprime Factorizations and Singular Perturbation*, Proceedings of The 5th Asian Control Conference, Melbourne, Australia, pp. 962-969, 2004.
- [7] Saragih, R. & Widowati, *Coprime Factor Reduction of Parameter Varying Controller*, International Journal of Control, Automation, and Systems, **6**(6), pp. 836-844, 2008.
- [8] Apkarian, P. & Adam, R.J., *Advanced Gain Scheduling Techniques for Uncertain System*, Proceeding American Control Conference, 1997.
- [9] Becker, G. & Packard, A., *Robust Performance of Linear Parametrically Varying Systems Using Parametrically-Dependent Linear Feedback*, System and Control Letters, **23**, pp. 205-215, 1994.
- [10] Bendotti, P. & Bodenheimer, B., *Linear Parameter Varying versus Linear Time Invariant Control Design for a Pressurized Water Reactor*, Research Article, California Institute of Technology, EE Dept., Pasadena, USA, 1998.
- [11] Bruzelius, F., *Linear Parameter Varying Systems*, PhD Dissertation, Departement of Signal and Systems, Chalmers University of Technology, Goteborg, Sweden, 2004.
- [12] Shin, J.Y., Balas, G.J. & Kaya, A.M., *Blending Methodology of Linear Parameter Varying Control Synthesis of F-16 Aircraft System*, NASA/CR-2001-211237, NASA Langley Research Center, 2001.
- [13] Spillman, M., *Robust Longitudinal Flight Control Design Using Parameter-Varying Feedback*, Journal of Guidance, Control and Dynamics, 2001.

- [14] Riyanto, B., *Dissipative Control of Linear Parametrically Varying Systems Using Output Feedback: an LMI Approach*, International Conference Modeling, Identification and Control, Austria, 1999.
- [15] Riyanto, B., *Self-Scheduled H_∞ Control Design for N250 Aircraft Dynamics Based on Polytopic LPV Model*, Proceedings of the IASTED International Conference Control and Applications, Banff, Canada, pp. 52-57, 1999.
- [16] Laban, M., *On-line Aircraft Aerodynamic Model Identification*, Ph.D Thesis, Delft University of Technology, 1990.
- [17] Mc. Lean, D., *Automatic Flight Control Systems*, Prentice-Hall, 1990.
- [18] Muhammad, H., Hariowibowo, H., Legowo, A. & Jenie, S.D., *Aerodynamic Parameter Identification of the N-250-PA1 Gatotkoco*, TR-01/1998/FTC-DT, IPTN Flight Test Center, Bandung 1998.
- [19] Etkin, B. & Reid, L.D., *Dynamic of Flight: Stability and Control*, Third Edition John Wiley & Sons, 1996.
- [20] Gahinet, P., Nemirovski, A., Laub, A. & Chilali, M., *The LMI Control Toolbox*, The MathWorks, Inc.: Natick, MA, 1995.