The All-DQ-Domain EMTP

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Abstract. This paper presents an improvement to dq-domain method of calculating electromagnetic transients. The proposed methodology works on dq-domain model for all components of the power system and during all time iterations. This is a new direction distinct from the old one where the network is invariably modeled in phase-domain. By modeling the network in dq-domain there is no more problem of interfacing machine to network as usually met in the existing method as machine is modeled invariably in dq-domain. Besides eliminating the time consuming transformation procedure between dq-domain to phase-domain or visa versa the new method is able now to fully exploit the infinite stability region of the trapezoidal rule of integration. The prediction/correction procedure of the conventional dq-domain method, which is notoriously known limiting the stability region, is no longer required. Comparing simulations using the new method and ATP, one of the conventional dq-domain version, show perfect conformity for small time step. For long time step while ATP is failing, the new method still converges accurately up to Nyquist's interval.

Keywords: *all-dq0-domain modeling; EMTP; infinite stability; multi-nodal method; trapezoidal rule.*

1 Introduction

Recent developments in the electromagnetic transients program (EMTP) has been directed to phase-domain modeling [1-4]. In [1], the machine phase-domain model is used motivated more by the need to take into account the saturation more rigorously. In [2-4], the choice is driven more by the weakness inherent in the old dq-domain procedure [5,6] which is based on prediction/correction procedure. This last procedure is used to get around the interfacing problem. In a way this procedure has been succeeded to solve the problem of the time varying of the phase-domain machine parameters elegantly but in the same time unfortunately it degrades the infinite stability region of the trapezoidal rule. This paper presents a new method based on dq-domain without resorting to that stability weakening procedure.

The solution approach almost common to any existing EMTP is based on Dommel multi-nodal method [5,6]. In the method, each lumped parameter is

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discretized individually using trapezoidal integration rule to create a linear relation between current and voltage at a given time t plus a known constant representing historical current/voltage at the time t-Δt. The set of all components current/voltage linear relations are assembled, according to Kirchoff's Law, in one equation, preferably in term of bus-conductance matrix as it offers sparsity that can be exploited to save space and to speed up calculation [7-10]. The resulting system of linear equations is solved for all bus voltages at time t. This procedure is iterated at every time step up to the end of simulation.

When all components are of non time-varying parameters, the resulting bus-conductance matrix needs to be assembled and decomposed only once at the initial stage. The situation will be different for component of nonlinear and/or time-variant parameters where the coefficient of the linearized current and voltage relation may change from one time step to another. Therefore, in order not to rebuild and refactor the conductance matrix at every time step, each nonlinear and/or time-variant component is represented separately as a current source using compensation procedure [11].

In the proposed method all components of the power system are modeled in dq0 domain in their respective individual discretizations as well as in their integration as a system of equations during all time cycle iterations. The main advantage of working in an all-dq0-domain procedure is its higher efficiency resulting from the space saving and less flops offered by a straightforward procedure in comparison to the phase-model.

The presentation of this paper will be organized as follows. In Section 2 the dq0 domain model of two main components of a power system, synchronous machine and lumped coupled balanced three-phase element, will be presented. Justification of the models, which is based on the Park's transformation theory, is well established [12-13]. The lumped coupled three phase element may represent the components of the network in general. In this paper, element of distributed parameter is not covered. Section 3 contains the description of the implementation of the new EMTP. The numerical simulation and discussions can be seen in Section 4. The paper conclusion is found in Section 5.

2 The All DQ0-Domain Modeling

2.1 Synchronous Machine In DQ0-Domain [6,12,13]

The following is the explanation of some symbols used in this paper. The symbols ψ, v, i stand for flux linkage, voltage and current and r and l for resistance and inductance, respectively. Subscripts *f* and *h* stand for the field and d-axis damper windings while subscripts g and k stand for the eddy current and q-axis damper windings respectively. Throughout this paper we use different type letters to indicate the designation of the symbols such as bold for vector or matrix, small for time domain variables or scalars and capital for phasors or admittances/impedances. We make distinction too between admittance/ impedance as a parameter of a component and as an element of the nodal matrix. The first is in small letter and the second is in capital.

By following motor convention and q-axis leading d-axis by 90º [13], the differential and algebraic equations governing the relationship of the machine terminal voltages, currents and flux linkages can be written in dq0-domain partitioned in stator and rotor block of variables as follows:

$$\left.\begin{aligned} \mathbf{r_s i_s} + \boldsymbol{\Omega\psi_s} + \frac{\mathbf{d\psi_s}}{\mathbf{dt}} &= \mathbf{v_s} \\ \mathbf{r_r i_r} + \frac{\mathbf{d\psi_r}}{\mathbf{dt}} &= \mathbf{v_r} \end{aligned}\right\} \tag{1}$$

where

$$\boldsymbol{\psi}_\mathbf{s} = \begin{bmatrix} \psi_d \\ \psi_q \\ \psi_0 \end{bmatrix}, \quad \mathbf{v_s} = \begin{bmatrix} v_d \\ v_q \\ v_0 \end{bmatrix} \quad \mathbf{i_s} = \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix}, \boldsymbol{\Omega} = \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{\psi}_\mathbf{s} = \begin{bmatrix} \psi_f \\ \psi_h \\ \psi_g \\ \psi_k \end{bmatrix}, \quad \mathbf{v_r} = \begin{bmatrix} v_f \\ v_h \\ v_g \\ v_k \end{bmatrix}, \quad \mathbf{i_r} = \begin{bmatrix} i_f \\ i_h \\ i_g \\ i_k \end{bmatrix}, \quad \boldsymbol{\psi}_\mathbf{s} = \begin{bmatrix} \psi_f \\ \psi_h \\ \psi_g \\ \psi_k \end{bmatrix},$$

$\mathbf{r_s}$ = Diag[$r_a \quad r_a \quad r_a$], $\mathbf{r_r}$ = Diag[$r_f \quad r_h \quad r_g \quad r_k$]

When saturation is neglected the flux linkage is a linear function of current which for a synchronous machine can be written in stator and rotor partitions as

$$\begin{bmatrix} \boldsymbol{\psi}_\mathbf{s} \\ \boldsymbol{\psi}_\mathbf{r} \end{bmatrix} = \begin{bmatrix} \mathbf{l_s} & \mathbf{m} \\ \mathbf{m^t} & \mathbf{l_r} \end{bmatrix} \times \begin{bmatrix} \mathbf{i_s} \\ \mathbf{i_r} \end{bmatrix} \tag{2}$$

where $\mathbf{l_s} = \mathbf{diag}\left(l_d, l_q, l_o\right)$,

$$\mathbf{l_r} = \begin{bmatrix} l_f & l_{fh} & 0 & 0 \\ l_{fh} & l_h & 0 & 0 \\ 0 & 0 & l_g & l_{gk} \\ 0 & 0 & l_{gk} & l_k \end{bmatrix} \text{ and } \mathbf{m} = \begin{bmatrix} l_{af} & l_{ah} & 0 & 0 \\ 0 & 0 & l_{ag} & l_{ak} \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

Here we focus only on the electromagnetic transients by assuming that the speed during short circuit is constant. This is quite reasonable in the case of a generator in no load. Even in a loaded case, the error may be very small for a short period between the inception and the clearance of the fault by the protection system. Nevertheless, a complete representation of the mechanical dynamic can easily be included.

Since all parameters in dq0-domain are time invariant then (1) can be rewritten as:

$$\begin{bmatrix} \mathbf{r_s} & \mathbf{0} \\ \mathbf{0} & \mathbf{r_r} \end{bmatrix} \begin{bmatrix} \mathbf{i_s} \\ \mathbf{i_r} \end{bmatrix} + \begin{bmatrix} \mathbf{u_s} \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{l_s} & \mathbf{m} \\ \mathbf{m^t} & \mathbf{l_r} \end{bmatrix} \begin{bmatrix} \mathbf{di_s/dt} \\ \mathbf{di_r/dt} \end{bmatrix} = \begin{bmatrix} \mathbf{v_s} \\ \mathbf{v_r} \end{bmatrix} \tag{4}$$

where : $\mathbf{u_s} = \Omega(\mathbf{l_s i_s} + \mathbf{m i_r})$ is the 'speed voltage' term.

It can be observed that the speed voltage term depends on $\mathbf{l_s}$ and $\mathbf{m}$, the machine parameters, so that in the reduction process it has to be excluded.

Therefore, for reduction purpose (4) can be rewritten in frequency domain as:

$$\left\{ \begin{bmatrix} \mathbf{r_s} & \mathbf{0} \\ \mathbf{0} & \mathbf{r_r} \end{bmatrix} + j\omega \begin{bmatrix} \mathbf{l_s} & \mathbf{m} \\ \mathbf{m^t} & \mathbf{l_r} \end{bmatrix} \right\} \begin{bmatrix} \mathbf{I_s} \\ \mathbf{I_r} \end{bmatrix} = \begin{bmatrix} \mathbf{V_s} \\ \mathbf{V_r} \end{bmatrix}$$

or concisely

$$\begin{bmatrix} \mathbf{Z_s} & \mathbf{Z_m} \\ \mathbf{Z_m^t} & \mathbf{Z_r} \end{bmatrix} \begin{bmatrix} \mathbf{I_s} \\ \mathbf{I_r} \end{bmatrix} = \begin{bmatrix} \mathbf{V_s} \\ \mathbf{V_r} \end{bmatrix} \tag{5}$$

where $\mathbf{Z_s} = \mathbf{r_s} + j\omega\,\mathbf{l_s}$, $\mathbf{Z_r} = \mathbf{r_r} + j\omega\,\mathbf{l_r}$ and $\mathbf{Z_m} = j\omega\,\mathbf{m}$

Machine is interfaced to the network through stator only, so the rotor equation in (5) is required to be represented to the stator side. To bring this about, $\mathbf{I_r}$ is eliminated from (5):

$$\left(\mathbf{Z_s} - \mathbf{Z_m Z_r^{-1} Z_m^t}\right)\mathbf{I_s} = \mathbf{V_s} - \mathbf{Z_m Z_r^{-1} V_r} \tag{6}$$

Finally, the machine admittance as seen from stator terminal amounts to

$$\mathbf{Y_{mach}} = \left(\mathbf{Z_s} - \mathbf{Z_m}\mathbf{Z_r^{-1}}\mathbf{Z_m^t}\right)^{-1} \tag{7}$$

To be noted, as $\mathbf{Z_r}$ and $\mathbf{Z_m}$ has a special nonzero structure and $\mathbf{Z_s}$ is diagonal then it can be shown that $\mathbf{Y_{mach}}$ is diagonal:

$$\mathbf{Y_{mach}} = \mathbf{diag}\left(y_d^{mach}, y_q^{mach}, y_0^{mach}\right) \tag{8}$$

2.2 Lumped Balanced Three-Phase Element in DQ0-Domain

Without losing generality network element is treated as lumped balanced three phase impedances. The equation of the voltage drop across a line in dq0 domain can be written

$$\left(\mathbf{r_{km}} + \mathbf{\Omega l_{km}}\right)\mathbf{i_{km}} + \mathbf{l_{km}}\,\mathbf{di_{km}}/\mathbf{dt} = \mathbf{\Delta v_{km}} \tag{9}$$

where

$$\mathbf{\Delta v_{km}} = \begin{bmatrix} \Delta v_d \\ \Delta v_q \\ \Delta v_0 \end{bmatrix}, \qquad \mathbf{i_{km}} = \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix}, \qquad \begin{array}{l} \mathbf{r_{km}} = diag\left(r_d\ r_d\ r_0\right) \\ \\ \mathbf{l_{km}} = diag\left(l_d\ l_d\ l_0\right) \end{array}$$

Again for reduction purpose after omitting the speed voltage $\mathbf{\Omega l_{km} i_{km}}$, (9) can be rewritten in frequency domain as:

$$\mathbf{Z_{km}}\ \mathbf{I_{km}} = \mathbf{\Delta V_{km}} \tag{10}$$

where $\mathbf{Z_{km}} = \mathbf{r_{km}} + j\omega\,\mathbf{l_{km}}$

In admittance form (10) can be rewritten

$$\mathbf{Y_{km}}\mathbf{\Delta V_{km}} = \mathbf{I_{km}} \tag{11}$$

Again it can be observed that as $\mathbf{r_{km}}$ and $\mathbf{l_{km}}$ are diagonal so $\mathbf{Z_{km}}$ or $\mathbf{Y_{km}}$ is.

To conclude this section, it can be stated that either machine or network element has its dq0-axis decoupled. An all-dq0-domain modeling was introduced for the first time in [14].

3 The Calculation Procedure

In this paper, the machine stator/rotor, the series and shunt component and the fault differential equations are discretized in dq0-domain. With the dq0-domain parameters of the machine, the series and shunt component are all of time-invariant parameters, discretization on their respective differential equations is required only once, that is before entering the time step cycle, hence the efficiency of the procedure. While for the fault, two cases may appear. First, in a balanced fault in which its dqo-domain admittances are constant so that they are required to be discretized once as above. Second, in an unbalanced fault where its dq0-domain admittances are time variant then they have to be discretized at every time step cycle.

3.1 The Formulation

3.1.1 The Discretization of Synchronous Machine

In this paper the synchronous machine is modeled in its full form as in (4). With a constant frequency assumption the speed voltage term $\mathbf{u}_s$ can be treated as a resistive voltage drop. Therefore, (4) now is rewritten as

$$\begin{bmatrix} \mathbf{r}_s + \boldsymbol{\Omega}\mathbf{l}_s & \boldsymbol{\Omega}\mathbf{m} \\ \mathbf{0} & \mathbf{r}_r \end{bmatrix} \begin{bmatrix} \mathbf{i}_s(t) \\ \mathbf{i}_r(t) \end{bmatrix} + \begin{bmatrix} \mathbf{l}_s & \mathbf{m} \\ \mathbf{m}' & \mathbf{l}_r \end{bmatrix} \frac{d}{dt} \begin{bmatrix} \mathbf{i}_s(t) \\ \mathbf{i}_r(t) \end{bmatrix} = \begin{bmatrix} \mathbf{v}_s(t) \\ \mathbf{v}_r(t) \end{bmatrix} \tag{12}$$

By using trapezoidal rule of integration (12) is discretized as

$$\begin{bmatrix} \mathbf{r}_{11} & \mathbf{r}_{12} \\ \mathbf{r}_{21} & \mathbf{r}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{i}_s(t) \\ \mathbf{i}_r(t) \end{bmatrix} = \begin{bmatrix} \mathbf{v}_s(t) \\ \mathbf{v}_r(t) \end{bmatrix} + \begin{bmatrix} \mathbf{hist}_s \\ \mathbf{hist}_r \end{bmatrix} \tag{13}$$

where

$$\mathbf{r}_{11} = \mathbf{r}_s + \left(\boldsymbol{\Omega} + \frac{2}{h}\right)\mathbf{l}_s\ , \qquad \mathbf{r}_{12} = \left(\boldsymbol{\Omega} + \frac{2}{h}\right)\mathbf{m}\,,$$

$$\mathbf{r}_{21} = \frac{2}{h}\mathbf{m}'\ , \qquad \mathbf{r}_{22} = \mathbf{r}_r + \frac{2}{h}\mathbf{l}_r\,,$$

$$\mathbf{x}_{11} = \mathbf{r}_s + \left(\boldsymbol{\Omega} - \frac{2}{h}\right)\mathbf{l}_s\,, \qquad \mathbf{x}_{12} = \left(\boldsymbol{\Omega} - \frac{2}{h}\right)\mathbf{m}\,,$$

$$\mathbf{r}_{21} = \frac{2}{h}\mathbf{m}'\,, \qquad \mathbf{x}_{22} = \mathbf{r}_r - \frac{2}{h}\mathbf{l}_r\,,$$

$$\mathbf{hist}_s = -\mathbf{v}_s(t-h) + \mathbf{x}_{11}\mathbf{i}_s(t-h) + \mathbf{x}_{12}\mathbf{i}_r(t-h)\,,$$

$$\mathbf{hist}_r = -\mathbf{v}_r(t-h) + \mathbf{x}_{21}\mathbf{i}_s(t-h) + \mathbf{x}_{22}\mathbf{i}_r(t-h)$$

Incorporation of (13) to the network equation is carried out directly in dqo-domain with rotor equation reduced.

The reduced equation is

$$\mathbf{r}_{\mathbf{s}}^{\mathbf{red}}\mathbf{i}_{\mathbf{s}}(t) = \mathbf{v}_{\mathbf{s}}(t) + \mathbf{hist}_{\mathbf{s}}^{\mathbf{red}} \tag{14}$$

where

$$\mathbf{r}_{\mathbf{s}}^{\mathbf{red}} = \mathbf{r}_{11} - \mathbf{r}_{12}\mathbf{r}_{22}^{-1}\mathbf{r}_{21}$$

$$\mathbf{hist}_{\mathbf{s}}^{\mathbf{red}} = \mathbf{hist}_{\mathbf{s}} - \mathbf{r}_{12}\mathbf{r}_{22}^{-1}\left\{\mathbf{v}_{\mathbf{r}}(t-h) + \mathbf{hist}_{\mathbf{r}}\right\}$$

Incorporation of the machine equation to network equation is easier in admittance form so the admittance version of (14) can be written as:

$$\mathbf{g}_{\mathbf{s}}^{\mathbf{red}}\mathbf{v}_{\mathbf{s}}(t) = \mathbf{i}_{\mathbf{s}}(t) - \mathbf{j}_{\mathbf{s}}(t) \tag{15}$$

where

$$\mathbf{j}_{\mathbf{s}}(t) = \mathbf{g}_{\mathbf{s}}^{\mathbf{red}}\mathbf{hist}_{\mathbf{s}}^{\mathbf{red}} \text{ and } \mathbf{g}_{\mathbf{s}}^{\mathbf{red}} = \left(\mathbf{r}_{\mathbf{s}}^{\mathbf{red}}\right)^{-1}$$

With an assumption that rotor field voltage is constant then (15) is ready to be incorporated to the network equation.

3.1.2 The Discretization of Network Branches

Individual branch equation (5) can be discretized straightforwardly as follows

$$\mathbf{r}_{\mathbf{km+}}\mathbf{i}_{\mathbf{km}}(t) + \mathbf{r}_{\mathbf{km-}}\mathbf{i}_{\mathbf{km}}(t-h) = \Delta\mathbf{v}_{\mathbf{km}}(t) - \Delta\mathbf{v}_{\mathbf{km}}(t-h) \tag{16}$$

$$\mathbf{r}_{\mathbf{km+}} = \left(\mathbf{r}_{\mathbf{km}} + \mathbf{\Omega}\mathbf{l}_{\mathbf{km}}\right) + \frac{\mathbf{l}_{\mathbf{km}}}{h} \tag{17}$$

$$\mathbf{r}_{\mathbf{km-}} = \left(\mathbf{r}_{\mathbf{km}} + \mathbf{\Omega}\mathbf{l}_{\mathbf{km}}\right) - \frac{\mathbf{l}_{\mathbf{km}}}{h} \tag{18}$$

Transforming (27) to admittance form we get

$$\mathbf{g}_{\mathbf{km+}}\Delta\mathbf{v}_{\mathbf{km}} = \mathbf{i}_{\mathbf{km}} - \mathbf{j}_{\mathbf{km}} \tag{19}$$

where

$$\mathbf{j}_{\mathbf{km}} = \mathbf{g}_{\mathbf{km+}}\left\{\mathbf{r}_{\mathbf{km+}}\mathbf{i}_{\mathbf{km}}(t-h) + \Delta\mathbf{v}(t-h)\right\} \text{ and } \mathbf{g}_{\mathbf{km+}} = \mathbf{r}_{\mathbf{km+}}^{-1}$$

3.1.3 The System Equation

After the machines and the branches of the power system having been discretized as shown above then by bringing them to their Norton equivalent, an integrated dq0-domain system of equations can be constructed in term of bus admittance matrix compactly as follows [15]:

$$\mathbf{G_{bus}V_{bus} = I_{bus}} \tag{20}$$

where $\mathbf{G_{bus}}$ is the bus admittance matrix, $\mathbf{V_{bus}}$ and $\mathbf{I_{bus}}$ are the voltage vector and the net source current vector respectively. The non-zero structure of $\mathbf{G_{bus}}$ is arranged to assume the non-zero structure of single phase case in which each entry will be consisting of a 3x3 sub matrix. While each entry of $\mathbf{V_{bus}}$ and $\mathbf{I_{bus}}$ will be consisting of a 3x1 sub vector that represents the dq0-axis voltages and net source currents. As the zero-axis circuit is fully decoupled from dq-axis circuits, a separate formulation can be envisaged to enhance efficiency. Source current at a bus is the net source currents incidence to the bus. They are representing the historical states of the branches and machines connected to the bus. By forcing the bus where the fault located to be numbered last (20) can be partitioned in group of faulted bus and healthy buses as follows:

$$\begin{bmatrix} \mathbf{G_{HH}} & \mathbf{G_{Hf}} \\ \mathbf{G'_{Hf}} & \mathbf{G_{ff}} \end{bmatrix} \begin{bmatrix} \mathbf{V_H} \\ \mathbf{V_f} \end{bmatrix} = \begin{bmatrix} \mathbf{I_H} \\ \mathbf{I_f} \end{bmatrix} \tag{21}$$

where $\mathbf{G_{HH}}$ is the sub matrix corresponding to the healthy buses, $\mathbf{G_{ff}}$ is the sub matrix corresponding to faulted bus, and $\mathbf{G_{Hf}}$ is the sub matrix coupling the healthy buses and the faulted bus. $\mathbf{V_H}$ and $\mathbf{I_H}$ are the vectors of the voltages and the net source currents of the healthy buses respectively, and $\mathbf{V_f}$ and $\mathbf{I_f}$ is the vectors of voltages and currents of the faulted bus respectively.

3.1.4 Incorporation of Fault

Matrix $\mathbf{G_{bus}}$ is constant at every time cycle while fault admittance matrix, as shown in the following is, except when balanced, ever changing from cycle to cycle.

Fault impedance is more natural to be represented in phase domain in input. The differential equation of a fault in phase domain can be written as follows

$$\mathbf{r}_{abc}^{fault}\mathbf{i}_{abc} + \mathbf{l}_{abc}^{fault}\frac{\mathbf{di}_{abc}}{\mathbf{dt}} = \mathbf{v}_{abc}^{fault} \tag{22}$$

Both parameters of the fault, $\mathbf{r}_{abc}^{fault}$ and $\mathbf{l}_{abc}^{fault}$, are time-invariant. However, to incorporate the fault into the system equation (20), which is in dq0-domain, it is more efficient to transform the fault equation from phase domain to dq0-domain rather than the other way round. The dq0 version of (22) is

$$\left(\mathbf{r}_{dq0}^{fault} + \mathbf{\Omega}\,\mathbf{l}_{dq0}^{fault}\right)\mathbf{i}_{dq0} + \mathbf{l}_{dq0}^{fault}\frac{\mathbf{di}_{dq0}}{\mathbf{dt}} = \mathbf{v}_{dq0}^{fault} \tag{23}$$

where

$\mathbf{r}_{dq0}^{fault} = \mathbf{C_P}\mathbf{r}_{abc}^{fault}\mathbf{C}_P^t$, $\mathbf{l}_{dq0}^{fault} = \mathbf{C_P}\mathbf{l}_{abc}^{fault}\mathbf{C}_P^t$, $\mathbf{i}_{dq0}^{fault} = \mathbf{C_P}\mathbf{i}_{abc}^{fault}$, $\mathbf{v}_{dq0}^{fault} = \mathbf{C_P}\mathbf{v}_{abc}^{fault}$ and $\mathbf{C_P}$ is the Park's transformation matrix.

When the fault is balanced both $\mathbf{r}_{dq0}^{fault}$ and $\mathbf{r}_{dq0}^{fault}$ are diagonal-matrix and time invariant [13-14] so that after having been discretized the fault equation can be incorporated straight- forwardly to (20), once at the initialization stage. Otherwise, those fault parameters are full-matrix and time-variant. Therefore, the fault incorporation to (20) has to be carried out at each time cycle.

The system equation of (20) when fault incorporated will change to:

$$\begin{bmatrix} \mathbf{G_{HH}} & \mathbf{G_{Hf}} \\ \mathbf{G'_{Hf}} & \mathbf{G_{FF}} \end{bmatrix}\begin{bmatrix} \mathbf{V_H} \\ \mathbf{V_f} \end{bmatrix} = \begin{bmatrix} \mathbf{I_H} \\ \mathbf{I_f} \end{bmatrix} \tag{24}$$

where

$$\mathbf{G_{FF}} = \mathbf{G_{ff}} + \mathbf{G_{fault}}$$

The most time consuming routine in the direct solution of (24) is the decomposition of the coefficient matrix of the linear system of equations. As the matrix is varying from cycle to cycle it is really tedious to decompose it at every cycle. The proposed algorithm offers a speedup by exploiting the special structure and characteristic of the coefficient matrix. The fault sub matrix $\mathbf{G_{FF}}$, being 3x3 which is generally small in comparison to the healthy sub matrix $\mathbf{G_{HH}}$, is the only time-varying sub matrix of $\mathbf{G_{Bus}}$. By designing a procedure using two block solutions, an efficient procedure can be obtained by allowing repeated decomposition only for a small part while the large part is decomposed only once at initialization stage.

The following is the equation for fault buses reduced from (24) by eliminating $\mathbf{V_H}$:

$$\left(\mathbf{G_{FF}} - \mathbf{G_{Hf}^{'}}\mathbf{G_{HH}^{-1}}\mathbf{G_{Hf}}\right)\mathbf{V_f} = \mathbf{I_f} - \mathbf{G_{Hf}^{'}}\mathbf{G_{HH}^{-1}}\mathbf{I_H} \tag{25}$$

After $\mathbf{V_f}$ having been obtained, $\mathbf{V_H}$ is then solved from

$$\mathbf{V_H} = \mathbf{G_{HH}^{-1}}\left(\mathbf{I_H} - \mathbf{G_{Hf}}\mathbf{V_f}\right) \tag{26}$$

Detailed steps to solve (25) and (26) are as follows:

I. Initialization stage:
   1. Decompose $\mathbf{G_{HH}}$
   2. Solve $\mathbf{G_{HH}X} = \mathbf{G_{Hf}^{'}}$ for $\mathbf{X}$
   3. Calculate $\mathbf{G_{HH}^{red}} = \mathbf{G_{Hf}^{'}X}$

II. Time cycle iteration
   At every time cycle do steps 4 to 8
   4. Solve $\mathbf{G_{HH}V_H^{temp}} = \mathbf{I_H}$ for $\mathbf{V_H^{temp}}$
   5. Calculate $\mathbf{I_f^{red}} = \mathbf{I_f} - \mathbf{G_{Hf}^{'}V_H^{temp}}$
   6. Calculate $\mathbf{G_{FF}^{red}} = \mathbf{G_{FF}} - \mathbf{G_{HH}^{red}}$
   7. Solve $\mathbf{G_{FF}^{red}V_f} = \mathbf{I_f^{red}}$ for $\mathbf{V_f}$
   8. Calculate $\mathbf{V_H^{final}} = \mathbf{V_H^{temp}} - \mathbf{XG_{Hf}^{'}V_f}$
   9. Stop

This cycle iteration has only an O(|U|) time complexity, where |U| is the number of non-zero factors of decomposed $\mathbf{G_{HH}}$, determined by the time expended at step 4, the most time consuming in the procedure.

4 Results and Discussion

4.1 The Simulation Data

As an illustration we simulate short circuit transients in a power system of five buses as shown in Figure 1. Data of the network are presented in Table 1. The three machines are identical with typical data in Table 2.

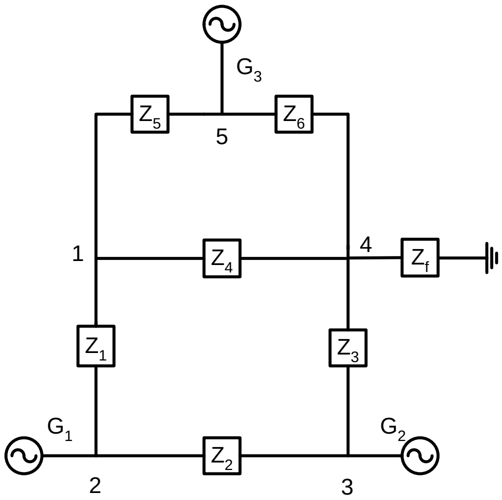


Figure 1 A five bus, three machines power system.

Table 1 Network data.

No.	*NS*	*NR*	*R(pu)*	*X(pu)*	Notes
Z_1	1	2	0.30	0.30	NS – sending end bus
Z_2	2	3	0.20	0.40	NR – receiving end bus
Z_3	3	4	0.10	0.10	R – resistance
Z_4	1	4	0.30	0.50	X – reactance at 60 Hz
Z_5	1	5	0.30	0.60	Base MVA – 100 MVA
Z_6	4	5	0.30	0.60	

All lines have the same zero-axis resistance and inductance. $R_0 = 2.0$ pu and $X_0 = 1.5$ pu .

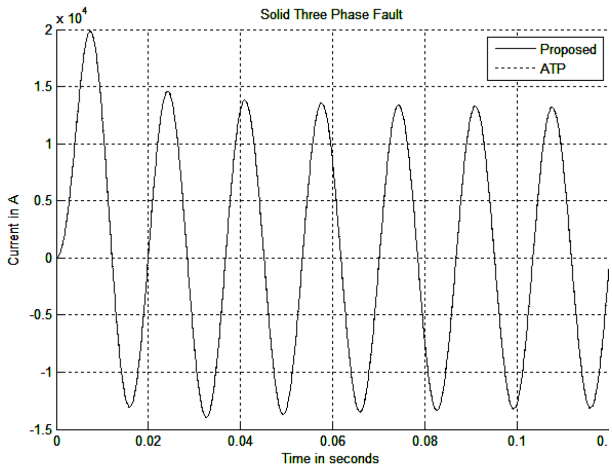
Table 2 Synchronous machine data.

L_f	6176.50	L_h	6044.76	L_{ak}	6044.76	R_a	0.1728
L_{af}	340.00	L_g	6133.89	L_k	6525.64	R_h	10.877
L_{fh}	6024.10	L_{ag}	228.43	L_o	0.7488	R_g	2.6471
L_d	10.31	L_{gk}	5733.78	R_o	0	R_k	52.312
L_{ah}	240.00	L_q	9.8496	R_f	0.3708		

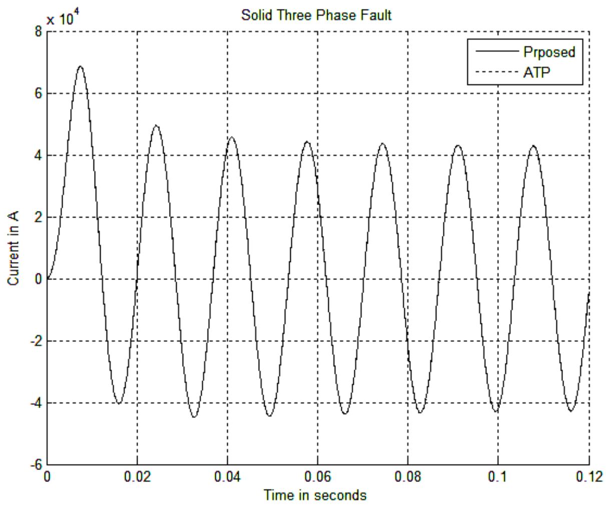
Note: All inductance values are in Ohm, evaluated at 60 Hz. Machine ratings: 100 MVA, 24 kV. Terminal voltage: 19.6 kV (maximum line to neutral).

4.2 The Simulation Results

The time simulations of the short circuit transients by using the proposed method are shown in Figure 2. Machines G_1, G_2 and G_3 are initially on no load. The fault is a solid three-phase short circuit at bus 4 and at $\theta_0 = 0$ for v_a. The time step is 0.0001 seconds. As a comparison the same problem is simulated using ATP [16] and the results are superposed in the same figures. The similarity of the results of the proposed method and ATP is almost perfect.



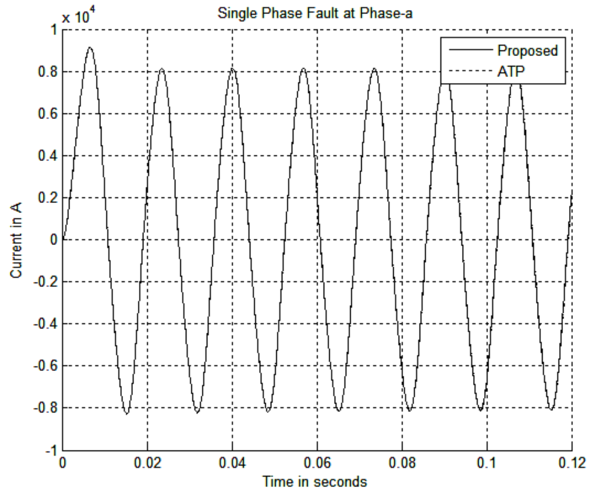
(a) Phase-a current at machine G_1.



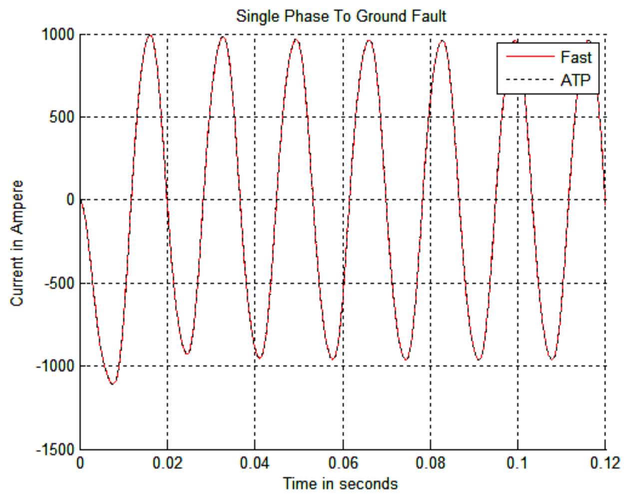
(b) Phase-a current at the fault.

Figure 2 Solid Three-Phase Fault at Bus 4.

In Figure 3 the plot of the solution for an unbalanced fault is presented. The fault is a solid phase-a to ground short circuit at $\theta_0 = 0$ of v_a. Again the simulation using ATP is superposed and it can be observed that visually there is almost no discrepancy of the currents in phase-a and phase-b in Figure 3(a) and Figure 3(b) respectively.



(a) Phase-a current at machine G_1

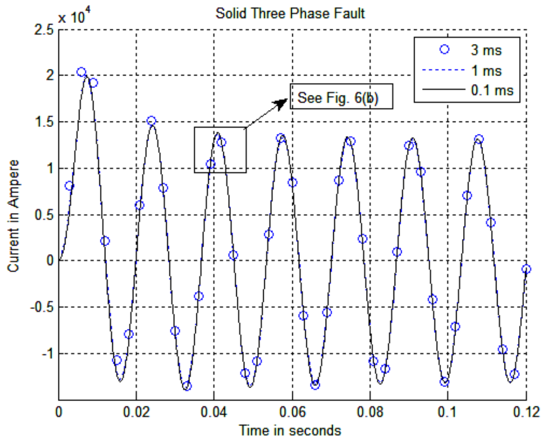


(b) Phase-b current at machine G1

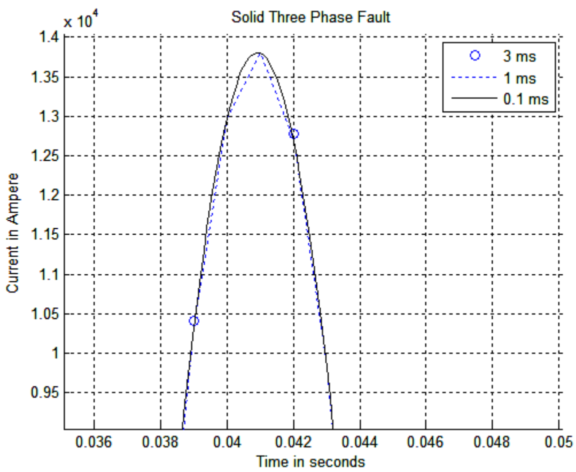
Figure 3 A solid phase-a to ground fault at Bus 4.

4.3 Accuracy and Convergence

The proposed method and ATP, for small step, give practically the same solution for currents and voltages. However, there is a big difference for long time step. In Figure 4 it is shown the plot of the simulation of the solid three phase fault for a range of time steps using the proposed method. There is no stability problem as found in ATP. In the latter the simulation always fails to converge when the time step is greater than 1 ms.



(a)



(b)

Figure 4 Plots for a range of time steps.

The proposed method, based on a genuine trapezoidal rule that has infinite regions of convergence [17], its solutions are always within reasonable accuracy even for long time step such as 3 ms, see Figure 4(a). Experimenting with very long time steps beyond the Nyquist interval (8.3 ms), has been giving a convergent solution although the plot of the result becomes meaningless as it loses important details, to tract the power frequency wave.

5 Conclusions

A new improvement to the dq-domain methodology of the electromagnetic transients program has been presented. The proposed procedure is based on all-dq0-domain formulation. The procedure has been shown effective to retain the infinite stability regions of a trapezoidal rule of integration while at the same time avoiding the time-consuming recurrent transformations between phase-domain and dq-domain. The unavoidable transformations in the case of the unbalanced fault have been managed to be constricted in a small part of the over-all procedure which incurs only a small portion of the total flops.

Appendix

Park's Transformation Matrix [13].

$$C_P = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & \sin\theta & 1/\sqrt{2} \\ \cos(\theta - \frac{2\pi}{3}) & \sin(\theta - \frac{2\pi}{3}) & 1/\sqrt{2} \\ \cos(\theta + \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) & 1/\sqrt{2} \end{bmatrix}, \text{ where } \theta = \omega t + \theta_0 .$$

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