GEOMETRIC PROPERTIES OF LANDSCAPES

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ICHTISAR.

Sifat-sifat geometri dari bentangalam telah diukur setjara kwantitas, seperti untuk kebanjakan usur-unsur daerah aliran dan beberapa mengenai gunung dan penjebaran dari daratan dan air. Hampir semua sifat daerah aliran didasarkan atas suatu sistim tata-atur aliran (stream ordering)

Dalam praktek telah dapat dipastikan beberapa hubungan antara unsur-unsur bentangalam masing-masing, seperti hukum-hukum djumlah sungai, pandjang sungai, terdjun sungai, luas daerah aliran, 'contributing areas'; 'constant of channel maintenance'; 'drainage density and texture ratio'; 'relief ratio'; 'hypsometric integral' untuk daerah aliran dan untuk gunung; 'frequency distribution' dari pandjang sungai; maksimum dari lereng lembah. Selandjutnja masih dapat diharapkan hasil-hasil jang menarik dari konsepsi 'geometrical similarity' dari daerah aliran serta dari teori pola penjebaran daratan dan air.

Peta-peta topografi Indonesia jang ketelitiannja sangat baik sudah memungkinkan penjelidikan geomorfologi kwantitatif disini. Sebagai permulaan penjelidikan mengenai 'drainage density and texture ratio', hypsometric integral', lereng lembah maksimum dan 'relief ratio' dapat dilakukan guna memperoleh angka-angka untuk membeda-bedakan bentangalam sebagai hasil dari iklim tropika lembab dan jang dibentuk oleh iklim sedang lembab/kering.

ABSTRACT.

Quantification of landforms has been executed for most watershed elements and adjacent slopes and some concerning mountain and distribution pattern of land and water. Almost all watershed characteristics are based on the system of stream ordering.

Empirically certain statistical relations of landscape elements have been proven to exist beyond doubt, like the laws of stream numbers, stream lengths, stream slopes, drainage basin areas, contributing areas; also: constant of channel maintenance; drainage density and texture ratio; relief ratio; hypsometric integrals for drainage basins and mountains; frequency distribution of stream lengths ; maximum valley side slopes. Furthermore, interesting results may be expected from the concepts of geometrical similarity of drainage basins, and the distribution pattern of land and water.

The good to excellent quality of Indonesian topographic maps invite an immediate investigation of drainage density and texture ratio, hypsometric integral, maximum valley side slopes, and relief ratio in order to obtain quantitative proofs in distinguishing landforms as products of the humid tropics from those of the temperate humid/dry climates.

INTRODUCTION

The important paper by Horton (1945) provided the science of landscapes, geomorphology, with means for kinematical analysis beside the usual geometric analysis. This evolution was made possible by quantification of landform elements —like length of streams, areas of drainage basins — both illustrating dimensional properties, and stream numbers representing a dimensionless feature of landforms. Quantification of landforms was already practiced before Horton's contribution in 1945; however, it was not done in a comprehensive way. Examples may be given like by himself (1932), Gilbert (1914), Juday and Birge (1941), Putnam (1917), Rubey (1933), Shaler (1899), and Shulits (1936).

Quantitative analysis of these dimensional and dimensionless properties of landforms during the past fifteen year has resulted in remarkable statistical relations, such as constant number of stream segment per unit basin area in given physical conditions, number of streams decreasing statistically with increasing stream order (to be defined). Statistical tests and recurrence of similar or equivalent relations in different areas under different physical conditions, e.g. climate and lithology among others, have proven the validity of these relations beyond doubt. In fact several students of quantitative geomorphology see morphometric laws in these relations (Strahler, 1957; Chorley, 1957 a).

At present most countries with advanced scientific knowledge practice quantitative geomorphic analysis, although the number of quantitative geomorphologists is still very small.

Next to Horton, Prof. A.N. Strahler of Columbia University, New York City, N.Y. in the United States has done much to promote the knowledge of quantitative geomorphology as is apparent from his work (Strahler, 1950, 1952 a, 1952 b, 1954 a, 1954 b, 1956, 1957, 1958) and his students' like Smith (1950, 1958), Schumm (1956 a, 1956 b, 1957), Morisawa (1957 a, 1957 b), and Melton (1958 a, 1958 b, 1959).
Other Americans are Langbein (1947), Chapman (1952), and Maxwell (1955). From Great Britain we know Chorley (1957 a, 1957 b, 1957 c, 1959, and 1961 with Carter), Drury (1951). From Japan Tanaka (1957) and Yoshikawa (1956) are known to have analyzed landforms in a quantitative way.
In the Netherlands Bakker and Le Heux (1946, 1947, 1950, 1952) contributed to landform quantification from a different angle than Strahler's School; still another approach is being nursed by Wickman (1962).

The practical value of quantitative geomorphology has been largely recognized, although some geomorphologists are still reluctant to give this modern development in landscape analysis its due credit (Thornbury, 1954). In the U.S.A. quantitative geomorphology is extensively used in studying practical problems of streams, like by Brush (1961), Hack (1960 a, 1960 b), Leopold (1953, 1956, 1957), Miller (1958), Wolman (1957). A well-deserved recognition may well be illustrated by the fact that at Columbia University research of quantitative analysis of erosional topography is sponsored by the Office of Naval Research, Geography Branch, Project NR 389 - 042. Quantitative geomorphology is now applicable in problems of soil erosion, sedimentation engineering, hydrology, and military science (Strahler, 1954 a).

By its quantitative and dynamic nature this modern extension of geomorphology leans heavily on statistics and hydrology beside the generally required knowledge of physics, chemistry, and mathematics of geologic education at Indonesian or American universities.

GEOMETRIC PROPERTIES MEASURED AND CALCULATED.

Certain relations between different dimensional and/or dimensionless properties of landscapes can be plotted as straight-line functions on semi-logarithmic or double logarithmic paper. Others are graphically represented by bar-graphs, or by a combination of concave, straight and convex curves — like hypsometric functions —; still others are adequately illustrated by mere algebraic functions.

In the following paragraphs some representative samples of morphometric relations are grouped into (1) straight-line functions on semi-log or log-log scales, (2) concave-straight-convex functions on arithmetic scales, (3) histograms, and (4) other graphical or algebraic representations.

Some subjects in the following discussion are only touched in a sketchy way, others are treated more extensively. For a more or less complete coverage of this subject the reader is referred to the cited original papers which are listed according to author at the end of this discussion.

This paper tries to give some insight of the achievements of the modern branch of geomorphology and to awaken interest among students of Indonesian geomorphology for the subject.

Stream orders. The basis of most geometric properties of drainage basins is the concept of stream ordering. On topographic maps the network of stream channels in a given water-shed is classed into orders of channel segments. Ordering is done as follows (see also fig. 1).

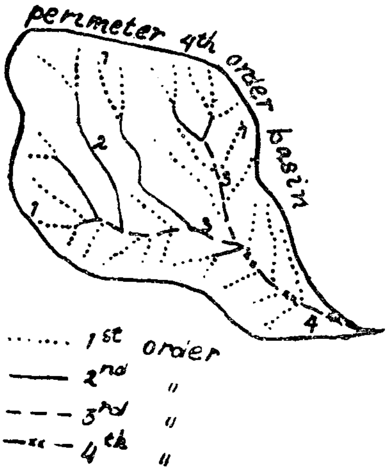


Fig. 1. The system of stream ordering. Basin 11h (K. Kandri, S. of Gunung Midangan, Lokulo region, C. Java). The largest part is underlain by hornfels, its northern quarter by marls. Scale 1 : 25.000.

Order	Number of stream segments	Length (km)	Total length (km)
1	31	6.9	10.4
2	7	2.1	
3	2	.9	
4	1	.5	

Basin perimeter (km) 3.9 Basin area (km²) .81
Basin relief (m) 325 Number of crenulations 19
Basin length, max. (m) 1600
Mean basin width (m) 700

The unbranched channel segments are given the order 1. Two or more first order segments join to form a channel segment of 2nd order; two or more 2nd order segments join into a 3rd order channel, and so on. A first order segment which join one of higher order does not change the order. The trunk stream of any given basin has the highest order, and this order is also assigned to the particular drainage basin.

The above given system of stream ordering has been advocated by Strahler (1954 a), which cancels out almost every subjective interpretation in selecting stream segments for ordering. It is preferred above the more subjective systems of Gravelius (1914) and Horton (1945).

For adequate accuracy Strahler (1954 a) has also advertised the use of large scale topographic maps of the order 1 : 24.000 or larger. Further on, all channel segments should be counted in the analysis, i.e. each inflection of contour line represent a stream channel, how small or dry it may be. This concept has been tested to be true (Morisawa, 1957 a).

The system of stream ordering proposes that order number is directly proportional to drainage basin dimension, channel size, and stream discharge at that place in the network. The dimensionless nature of order number facilitates comparing drainage basins of different scale (Strahler, 1957).

I. Straight-line functions on semi-log or log-log scales.

Law of stream numbers. Horton (1945) found that the numbers of stream segments of each order form an inverse geometric series with order number. Numerous investigations has proven this law to be valid (Strahler, 1952; Schumm, 1956 a).

Figure 2a illustrates this law for the drainage basin of figure 1 in the Lokulo area, Central Java. The number of streams of each order on the logarithmic ordinate is plotted against the linear abcissa designating stream order. The straight-line plot obtained for this case is fitted by inspection and is represented by the general formula:

$$\text{Log } N_0. = a - bo \ldots\ldots\ldots\ldots \qquad (1)$$

Where N_0 is number of streams of order o, a is a constant, b represents the slope of the line or also called regression coefficient, and o is the particular order. The anti-logarithm of b is equivalent to Horton's (1945) *bifurcation ratio* r_b. The bifurcation ratio is the ratio between number of stream segments of a given order to the number of segments of the next higher order. It has been found that the bifurcation ratio is a very stable number from region to region, except where geologic controls dominate (Strahler, 1957).

Law of stream lengths. Horton (1945) defined the Law of Stream-lengths as "the average lengths of streams of each of the different orders tend closely to approximate a direct geometric series in which the first term is the average length of stream of the first order" (fig. 2b). The mathematical function is

$$\text{Log } \overline{L}_0 = \text{Log } \overline{r}_1 \text{ o} - c \ldots\ldots\ldots\ldots \qquad (2)$$

where $\overline{Lo}$ is mean length of streams of order o, $\overline{r}_1$ is the mean length ratio (which is the mean ratio of the mean stream lengths of one order to those of the next lower order), o is the particular order, and c is a constant.

Empirically it was found that the above said relation rarely is a straight line. This fact is probably due to Strahler's system of stream ordering, which differs from Horton's (1945) and Chorley's (1957 a). Therefore, Strahler (1957) suggested to plot the logarithms of total stream lengths of each order ($\text{Log } \Sigma \, l_0$) against the logarithms of stream orders (Log o). Here a straight-line plot becomes apparent as an inverse logarithmic sequence (Fig. 2c).

Law of stream slopes. Again Horton (1945) suggested that slope of streams and stream order are related by an inverse geometric series. Chorley (1957 a) found this to be largely true for the three areas he studied.

Law of drainage basin areas. Schumm (1956 a) found that drainage basin areas increase with increasing master stream (i.e. basin) order according to a direct geometric series. This law has been verified, among others by Chorley (1957 a).

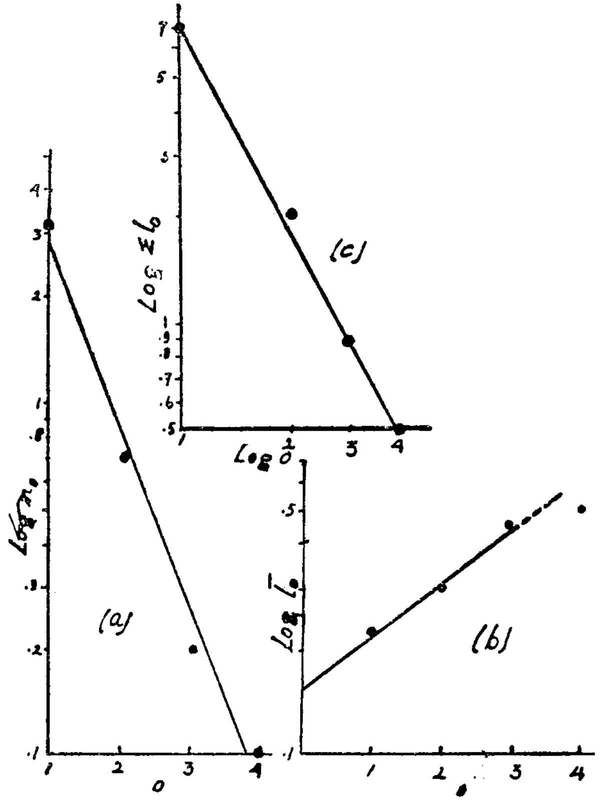


Fig. 2. a. Law of stream numbers; N_0 is number of streams of order o, o is stream order.
b. Law of stream lengths; $\bar{l}_0$ is average stream length of order o, o is stream order.
c. Revised law of stream lengths; $\Sigma\ l_0$ is total stream lengths of order o, o is stream order.

Law of contributing areas/Constant of channel maintenance. Schumm (1956 a) also noted a relation between drainage basin areas and the sum of the channel lengths which drain these contributing areas. Strahler (1957) modified this law and Chorley (1957 a) coined the term for this relation as Schumm's Law of Contributing areas' which is: "The relationship between drainage basin areas of each order and the total stream lengths contained within and supported by these areas is a direct logarithmic function, the regression coefficient of which is unity, and the value of the area when the total stream length is unity equals Schumm's (1956 a) "Constant of Channel Maintenance".

Chorley (1957 a) found in three regions of maturely dissected sandstone terrain without pronounced structural control, that their geometry is indicated by uniform, dimensionless ratios. This fact was discovered when he plotted the mean areas of basins against the mean lengths of corresponding second, third, and fourth order streams for regions on log-log scale. The straight-line plot is given by the formula:

$$\mathrm{Log}\,\overline{A}_o = c + b\,\mathrm{Log}\,\overline{l}_o \;\ldots\ldots\ldots\ldots\ldots \qquad (3)$$

where $\overline{A}_o$ is mean basin area of order o, c is a constant, b indicates the slope of the line, and $\overline{l}_o$ is mean stream length of order o.

Relief ratio. Schumm, (1956a) defined the relief ratio as the ratio between total basin relief, i.e. difference in elevation of basin mouth and summit, and basin length, measured as the longest dimension of the drainage basin. The relief ratio thus indicates the overall slope of a drainage basin. Schumm (1954) illustrated the value of relief ratio with sediment loss. By plotting mean annual sediment loss in acre feet per square mile against the relief ratio for a variety of small drainage basins in the Colorado Plateau province, a more or less straight line relation was found (Fig. 3).

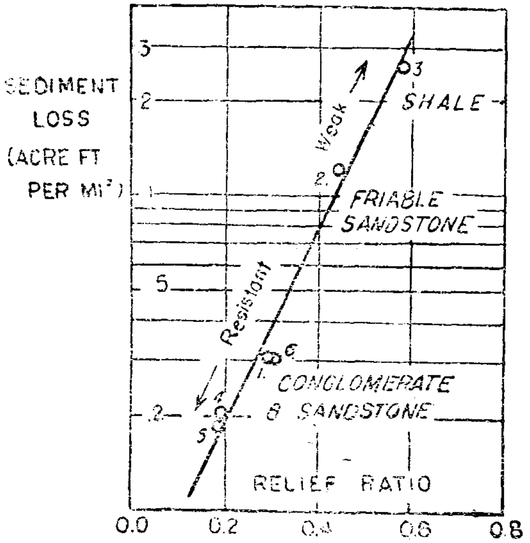


Fig. 3. Regression of sediment loss on relief ratio, Schumm (1954):

Drainage density and texture ratio. Smith (1950) has elaborated on the term drainage density which has been defined by Horton (1945) as the average length of streams within the basin per unit area ($D_d = L/A$). Smith added a property to erosional topography which he termed texture ratio: "The ratio of the number crenulations N on that contour having the maximum number within a given drainage basin to the length (in miles) of the perimeter P of the basin ($T = N/P$)".

Plotting drainage density (ordinate) against texture ratio (abcissa) on log-log paper, he obtained a straight-line plot which is given by the general formula:

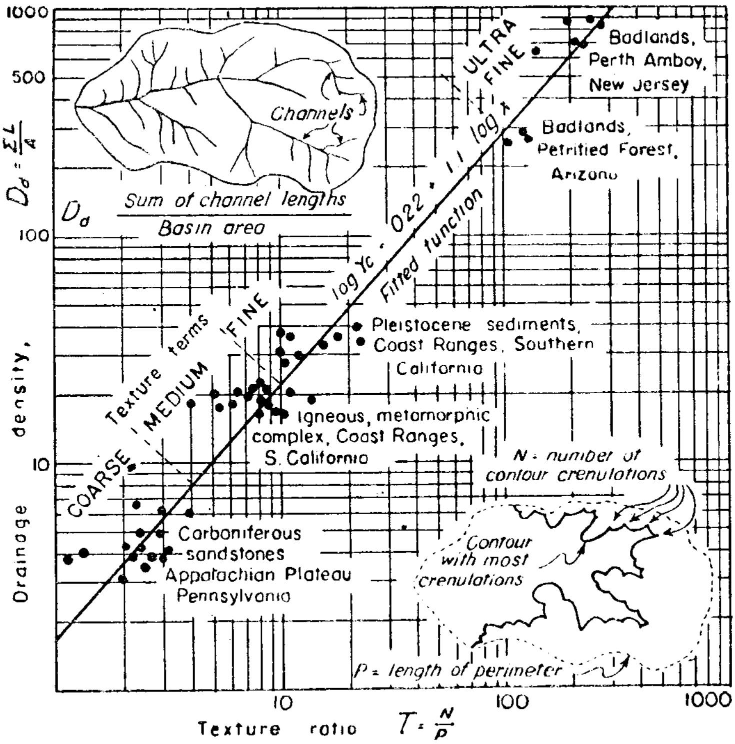


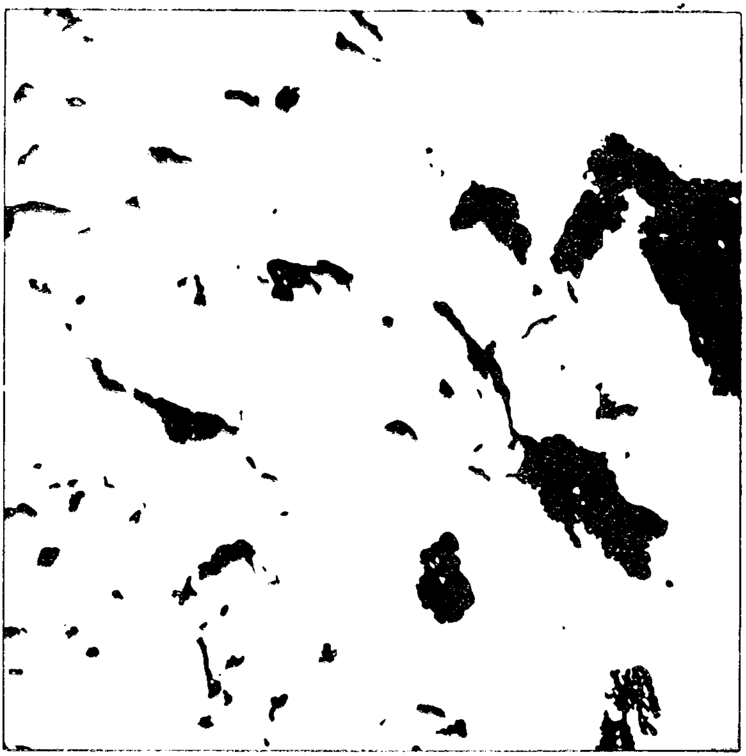
Fig. 4. Drainage density and texture ratio, Strahler (1954 a).

$$\text{Log } Y_c = \text{Log } a + b \text{ Log } X \qquad (4)$$

In his first study Smith (1950) found the function to be Log Y_c = .220 + 1.11 Log X. In the Badlands of South Dakota (Smith, 1958) this function is Log Y_c = .225 + 1.15 Log X. The marked clustering on the graph of D_d and T enables Smith to assign numbers to coarse, medium, fine, and ultra-fine textures of topography, which are respectively less than 4.0, 4.0 - 10.0, 10.0 - 100.0, and more than 100.0 (Smith, 1950, 1958).

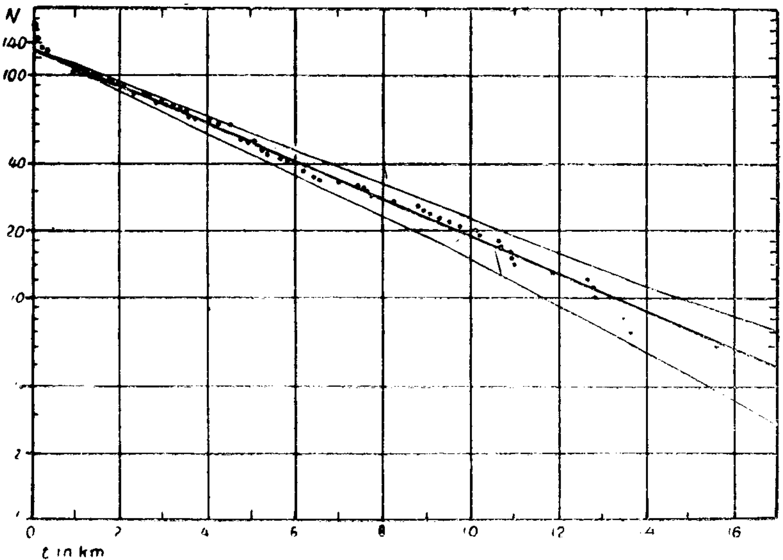
Distribution pattern of land and water. Recently Wickman (1962) began in a statistical way to analyze the distribution pattern of land and water in the glacial-lake country of Eastern Sweden (Fig. 5).

He made use of topographic maps of the scale 1 : 50.000 across which parallel lines, mostly in E - W direction, are laid at regular intervals. Along these lines distances are measured between one shore and the next across land and across water. Distances of each type are classified according to length and are listed as number of distances larger than a value t kilometers. Plotting number of distances on the logarithmic ordinate against linear



Map showing the distribution of land and water on the topographic map "Hofors So" Sweden. Original scale 1 50,000 Reproduction scale 1 20,000 Black water, white land (Wickman, 1962.)

Fig. 5



(Wickman 1962)

Shore distance curve for land on the topographic map "Hofors So" scale 1 50.000 Direction of measured lines E-W values of distances on the abcissa generally yields a straight-line plot. In figure 6 the heavy line depicts the straight-line relation and the light curves show the standard deviation for this line.

Wickman (1962) is of the opinion that this analysis might be valuable in the formulation of a theory concerning the relationships between tectonics, sediments, and other relevant factors. He proposed a further analysis on scales different than the one used in this study and for different regions of the world.

II. Concave-straight-convex curves on arithmetic scale.

Hypsometric integral. Strahler (1952) and his students (Miller, 1953; Schumm, 1956; Coates, 1956) applied the hypsometric analysis — i.e. the relation of horizontal cross-sectional drainage basins area to elevation —to numerous drainage basins of low order. Strahler (1957) described the method as follows:

"Figure 8 (Fig. 7 in this discussion) illustrates the definition of the two dimensionless variables involved. Taking the drainage basin to be bounded by vertical sides and a horizontal base plane passing through the mouth, the relative height is the ratio of height of a given contour h to total basin height H. Relative area is the ratio horizontal cross-sectional area a to entire basin area A. The percentage hypsometric curve is a plot of the continuous function relating relative height y to relative area x.

As the lower right-hand diagram of Figure 8 (Fig. 7) shows, the shape of the hysometric curve varies in early geologic stages of development of the drainage basin, but once having attained an equilibrium, or mature stage (middle curve on graph), tends to vary little thereafter. Several dimensionless attributes of the hypsometric curve are measurable and can be used for comparative purposes. These include the integral, or relative area lying below the curve, the slope of the curve at its inflection point, and the degree of sinuosity of the curve. Many hypsometric curves seem to be closely fitted by the model function shown in the lower left corner of Figure 8 (Fig. 7) although no rational or mechanical basis is known for the function.

Now that the hypsometric curves have been plotted for hundreds of small basins in a wide variety of regions and conditions, it is possible to observe the extent to which variation occurs. Generally the curve properties tend to be stable in homogeneous rock masses and to adhere generally to the same curve family for a given geologic and climatic combination."

Chorley and Morley (1959 a) have advanced a simple solution in approximating the hypsometric integral through mathematical analysis.

Yoshikawa (1956) applied hypsometric analysis to mountain regions and

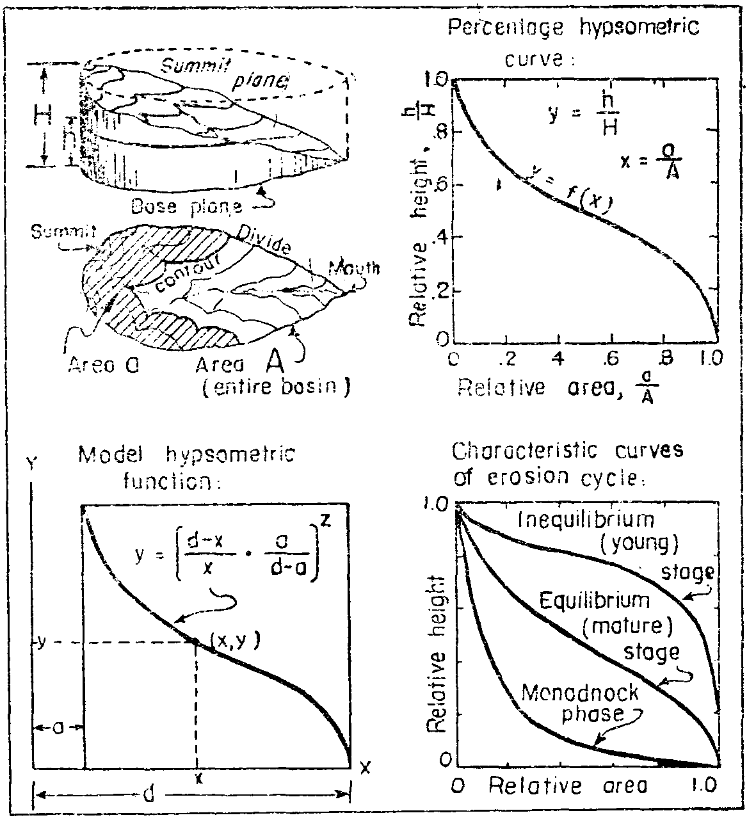


Fig. 7. Method of hypsometric analysis (Strahler, 1957).

found some parameters indicative of geomorphic age and the number of mountain summits.

III. Histograms.

Frequency distribution of stream lengths. Length of stream channel generally illustrates the scale of other units of a drainage basin. For certain drainage basins Schumm (1956 a) studied their stream lengths according to their frequency distribution (Fig. 8). It has been found that stream lengths are skewed right, but by using the logarithm of length the skewness can be largely corrected. Strahler (1954 b) used arithmetic mean, estimated population variance, and standard deviation to describe different drainage networks.

IV. Other representations.

Dimensional analysis and geometrical similarity. For correlation studies of drainage basins which only differ in size but are geometrically similar, Strahler (1954 a) proposed an ingenious method. Figure 9 illustrates the principles of the concept of similarity. In the following section we may cite Strahler's (1957, p. 913-913) own explanation.

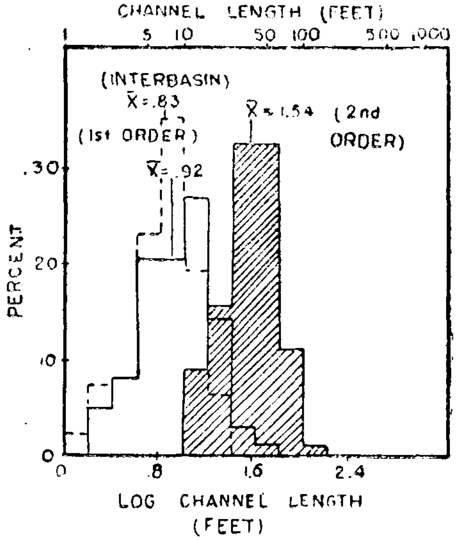
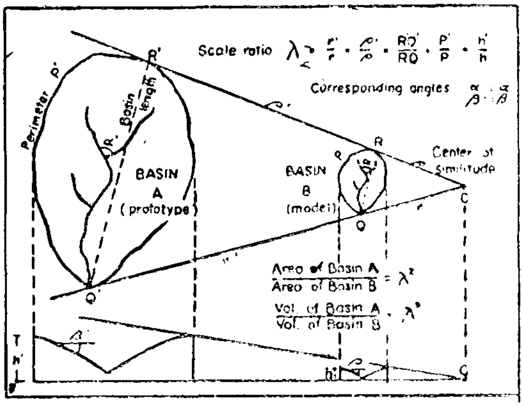


Fig. 8. Histogram of the logarithms of stream channel lengths. (Schumm, 1956 a).



Principles of dimensional analysis and geometrical similarity applied to drainage basins (Strahler 1957)

Fig. 9. Illustrates the principles of the concept of geometrical similarity

"Basins A and B are assumed to be geometrically similar, differing only in size. The larger may be designated as the prototype, the smaller as the model. All measurements of length between corresponding points in the two basins bear a fixed scale ratio, λ. Thus, if oriented with respect to a common center of similitude, the basin mouths Q′ and Q are located at distances r′ and r, respectively from C; the ratio of r′ to r is λ. In short, all corresponding length measurements, whether they be of basin perimeter, basin length or width, stream length or relief (h′ and h in lower profile), are in a fixed ratio if similarity exists.

All corresponding angles are equal in prototype and model (figure). This applies to stream junction angles α′ and α, and to ground slope angles β′ and β. Angles are dimensionless properties; hence the generalization that in two geometrically similar systems all corresponding dimensionless numbers or products describing the geometry must be equal.

Studies of actual drainage basins in differing environments show that in many comparisons in homogeneous rock masses, geometrical similarity is closely approximated when mean values are considered, whereas in other comparisons, where geologic inhomogeneity exists, similarity is definitely lacking. One advantage of using the principles of similarity as a basis of operation is that it focusses attention upon (a) linear scale differences that are independent of form or shape properties, and (b) form differences existing independently of size differences.

Maximum valley side slopes. Maximum valley side slopes are measured at intervals along the steepest slopes of valley walls from divides to the local stream channels. In numerous areas of different geology and climate it has been found that intra-areal variance of maximum valley side slopes is relatively small compared with inter-areal differences. This fact may be a valuable mean to calculate sediment production (Strahler, 1957).

Slope maps. Strahler (1956) devised a method to determine slope conditions in a drainage basin. Following Strahler's 1957, p. 918) discussion on the subject:

"(1) A good topographic map is taken. (2) On this map the slopes of a short segment of line normal to the trend of the contours is determined at a sarge number of points. These may be recorded as tangents or sines, depending upon the kind of map desired. (3) These readings are contoured with lines of equal slope, here called isotangents. (4) The areas between successlive isotangents are measured with a planimeter and the areas summed for each slope class. (5) This yields a slope frequency percentage distribution. Because the entire ground surface has been analyzed, the mean, standard deviation, and variance are treated as population parameters, at least for purposes of comparision with small samples taken at random from the same area.

Lines of equal sine of slope or isosines, may also be drawn. The interval between isosines on the map becomes the statistical class on the histogram. Sine values are designated as g values because the sine of slope represents that proportion of the acceleration of gravity acting in a downslope direction parallel with the ground surface."

Drumlin geometry. Reed et al. (1962) found in quantitative studies on drumlins that these landforms approximates ellipsoids in shape and show a normal distribution of orientation within small areas.

Chorley (1959 b) related the plan-shape of drumlins with lemniscate loops; generally in a succesfull way.

CONCLUSION

The quantification of landforms has proven its worth during the past fifteen year since Horton's (1945) comprehensive formulation of quantitative geomorphology. Numerous aspects of watershed morphology, slopes, channel segments, stream orders, junction angles of streams, topographic texture, drainage density, etc. have received most of the attention, particularly through the leadership of Strahler's school. Other aspects of quantitative geomorphology have been tackled at a smaller scale, like the hypsometric analysis for mountains, the distribution pattern of land and water, and drumlin geometry.

Most morphometric studies require good topographic maps at an adequate scale of 1 : 50.000 or larger. Quantitative morphological studies in Indonesia may be practiced immediately by virtue of the good to excellent character of its topographic maps (Pannekoek, 1946). In this way we may probably arrive at figures for differentiating landforms of the humid tropics from those of the humid/dry temperate climates, where most of the known morphometric investigations have been executed until now (Tjia, 1962). Interesting prospects seem to be promised by studies of drainage densities, texture ratios, hypsometric integrals, maximum valley side slopes, and relief ratios in Indonesia.

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